Evolution of urban structures

Some results coming from the analysis of historical sources

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Introduction

Towards a quantitative description of cities, their formation and evolution

Focus on fundamental aspects:

- Understanding quantitatively the main mechanisms of urbanization
- Modeling the evolution of urban systems

Increasing availability of historical urban data

- Mobility, infrastructures, transportation systems…
- In particular: transportation networks shape the city (historical maps)
- From directories => Evolution of economic activities (Julie Gravier)

Not a new goal

- Long standing, interdisciplinary effort ! Quantitative geography (1960s)
- Morrill (1965) Stochastic model of road network evolution
- Historical approaches
  - Cellular-automaton
  - Percolation, DLA
  - Urban economics

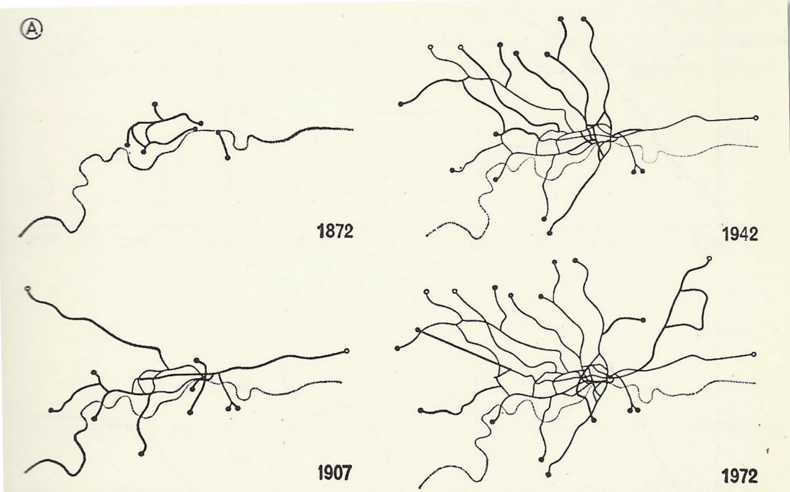


Fig. 5.7. Stages in the growth of the London underground rail network. Source: Cope, 1967.

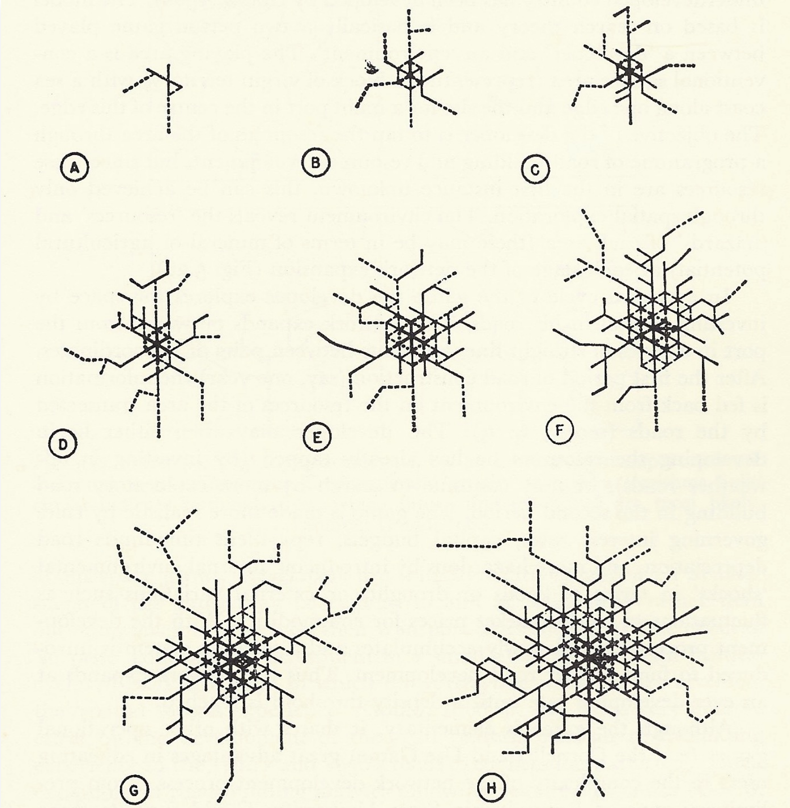


Fig. 5.25. Simulation of road network around a central node by Monte Carlo procedures. Source: Morrill, 1965.

From historical sources to the evolution of spatial networks: Illustration on case-studies

- Evolution of road networks: elementary processes
  Groane, Italy; 1833-2007

- Evolution of road networks: importance of planning
  Paris, France; 1789-2010

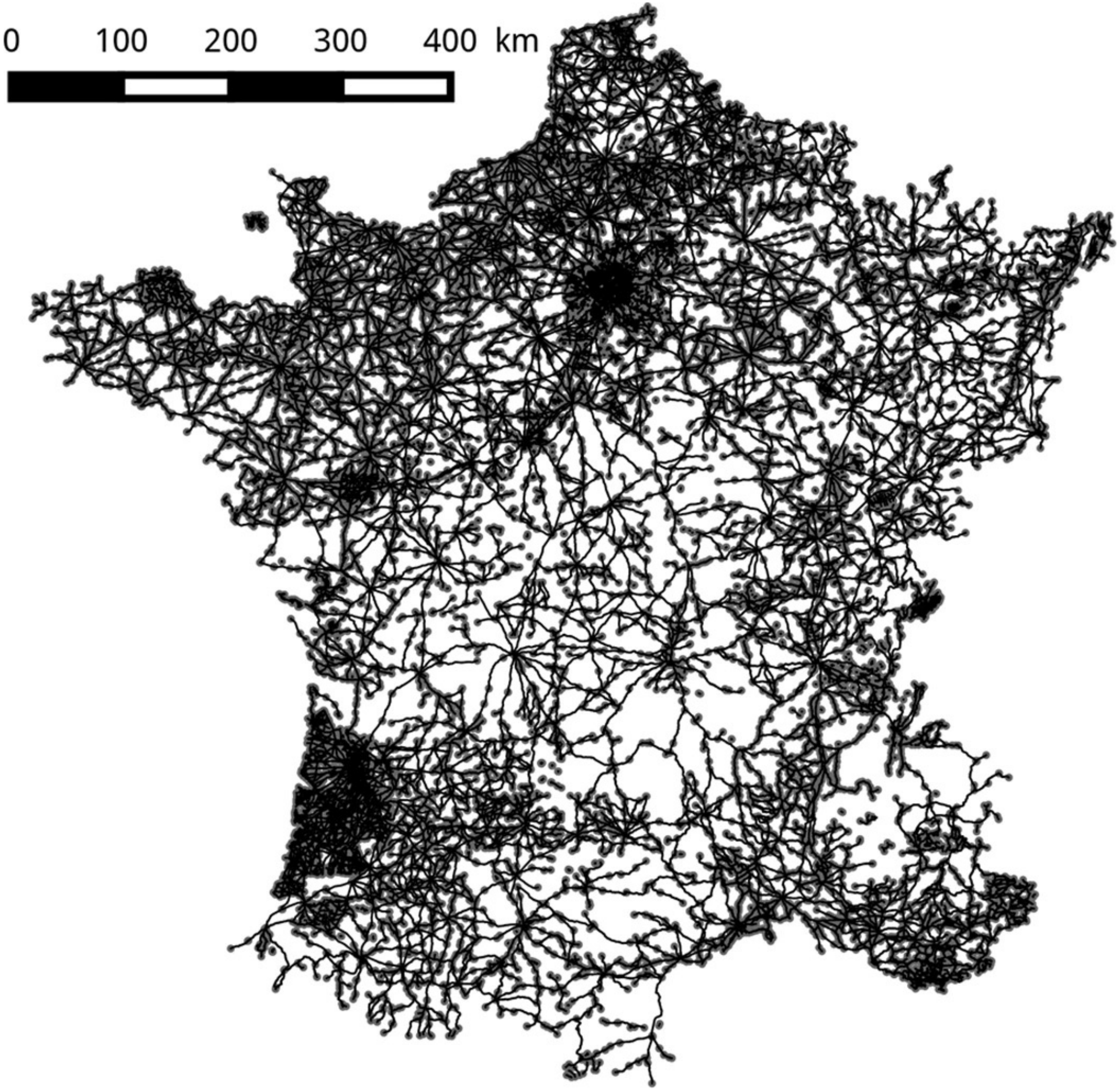
- Evolution of subway networks: universal features
  World subway networks 1863-20

- Other studies: Cassini map; evolution of the French railway

Road network

Map of France
(Cassini, 18th)
Collaborative
digitalization and
validation

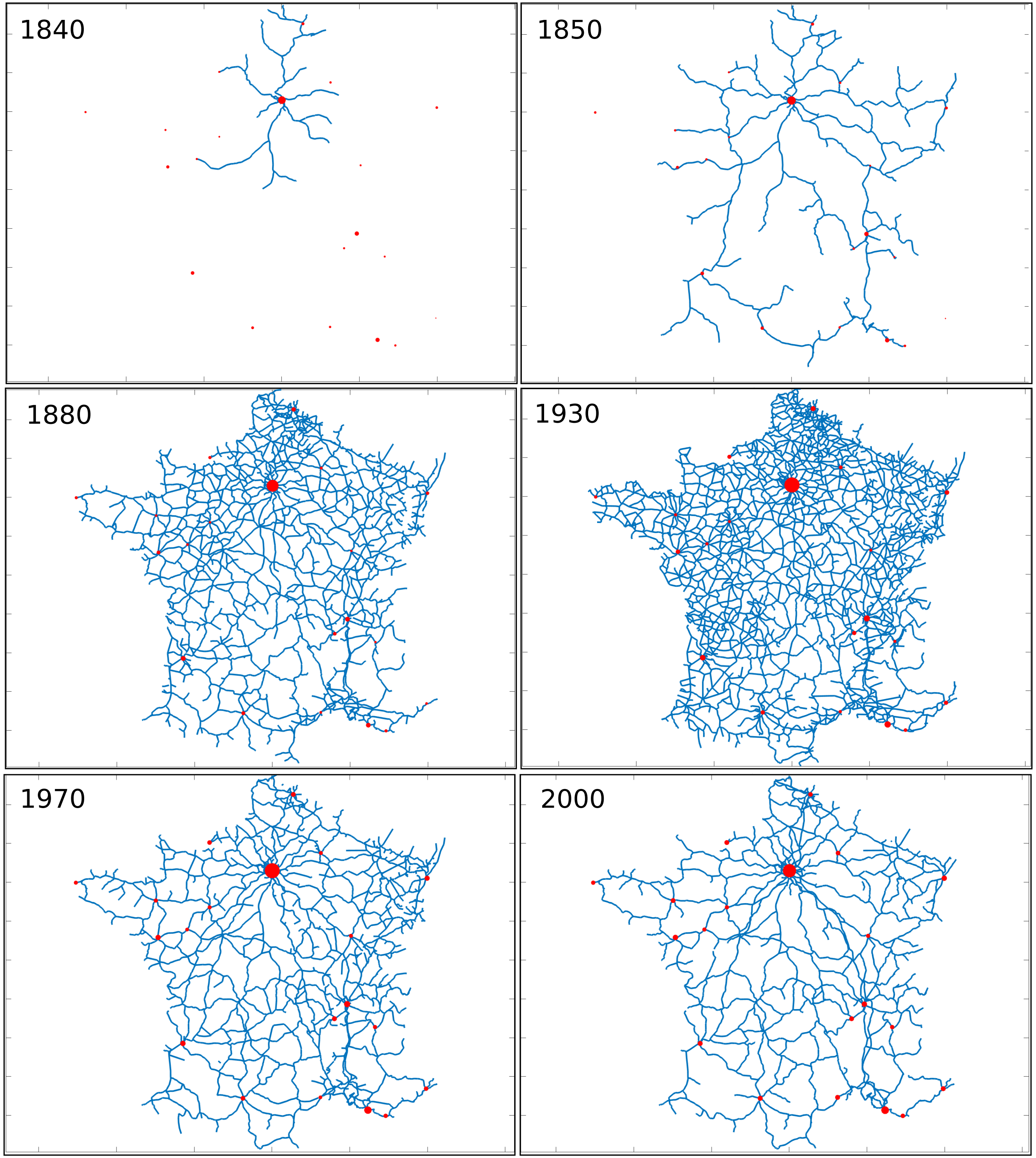
"Building Inspector"
Developed by
NYPL and used
for the validation
of buildings
automatically vectorized
from insurance maps.



Perret, Gribaudi, MB, Scientific Data 2015

French railway evolution 1840-2000

Data:
Railway: Thomas Thevenin
Population data:
www.geohistoricaldata.org



Bottinelli, Gherardi, MB 2019

Evolution of road networks: Elementary processes

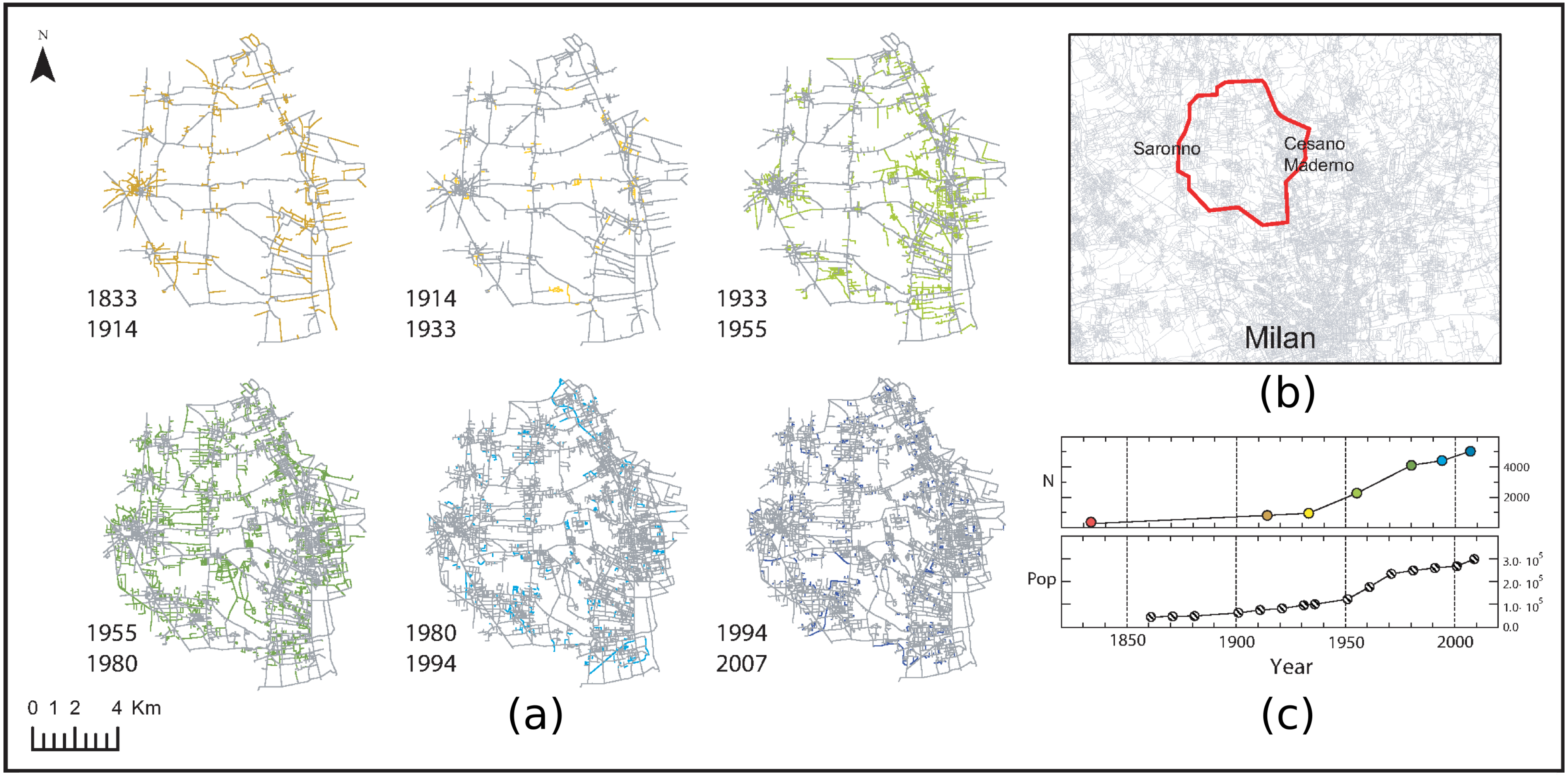
Strano, Nicosia, Latora, Porta, MB

Nature Sci. Rep. (2012)

Road network evolution: Groane region (Italy)

Date	Source	Owner	Format
1833	Topographical Map of Lombardy-Venetia Kingdom	Italian Military Geographic Institute	Raster
1914	Map of Italy	Italian Military Geographic Institute	Raster
1933	Map of Italy	Italian Military Geographic Institute	Raster
1955	Aerial Photography Survey	Italian Military Geographic Institute	Raster
1980	Lombardy Regional Map	Lombardy Region	Raster
1994	Lombardy Regional Map	Lombardy Region	Raster
2007	Mosaic of Urban Municipalities Plans	Lombardy Region	Vectorial

Road network evolution: Groane region (Italy)



Betweenness centrality (Groane 1833-2007)

Backbone of stable central roads

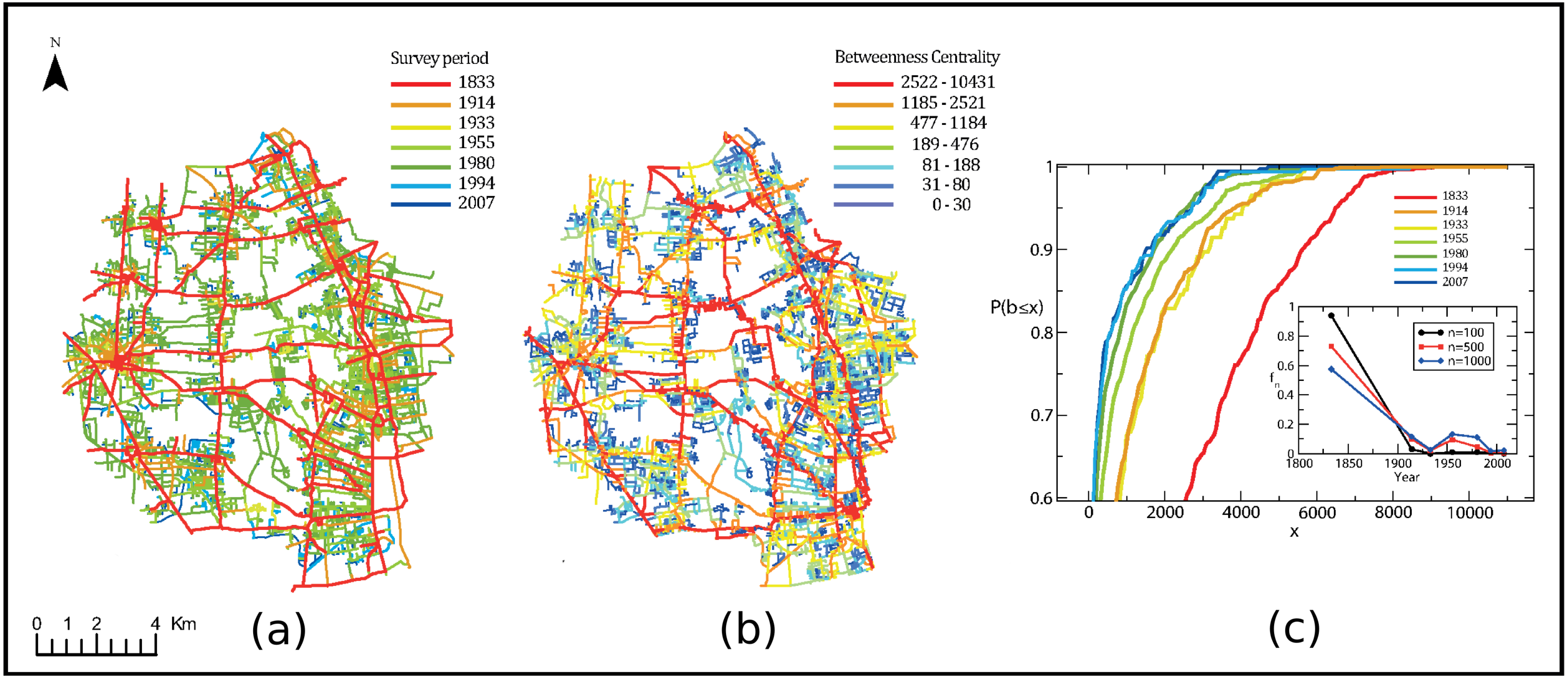
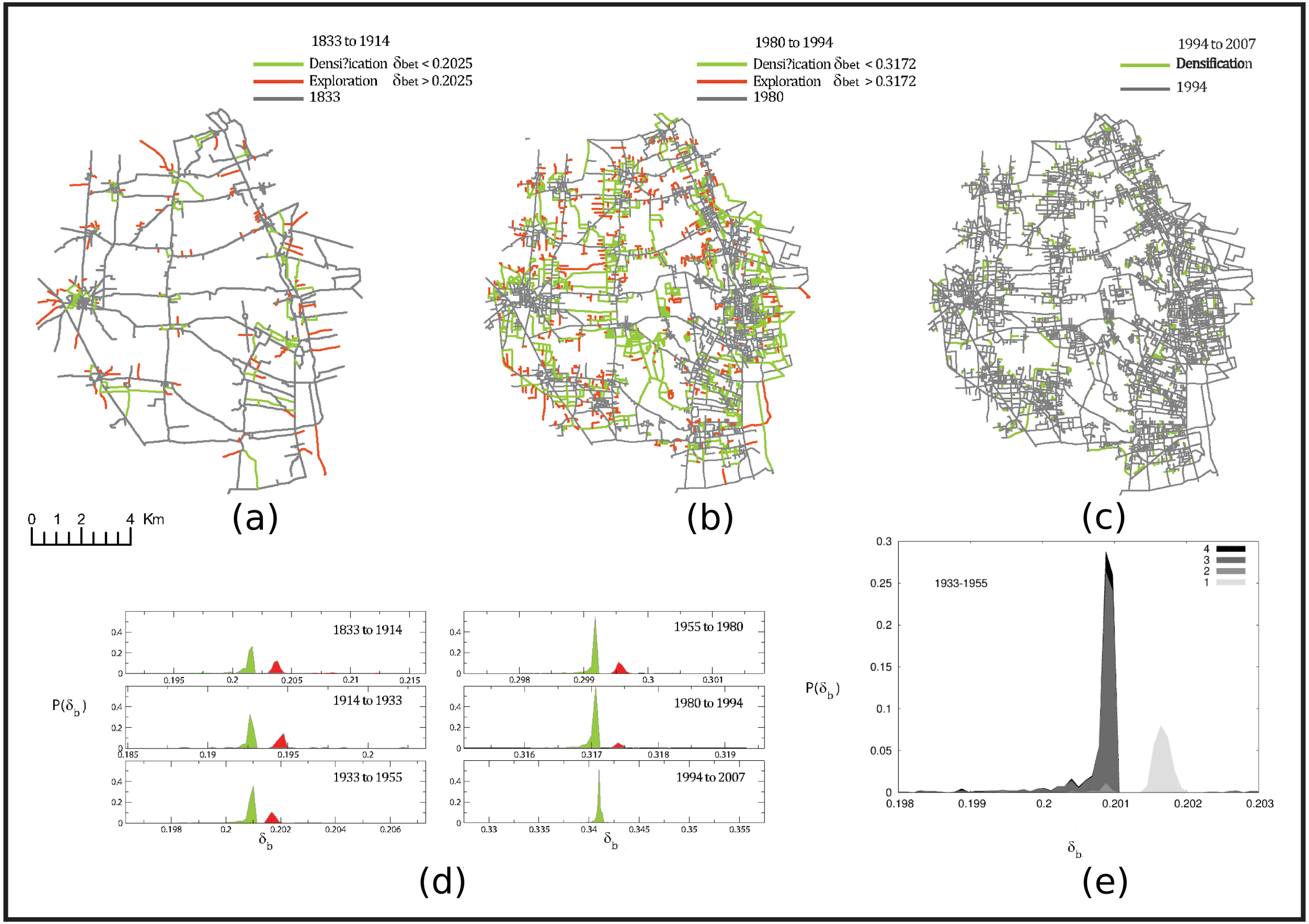


Figure 6 | Color maps indicating (a) the time of creation of each link and (b) its value of betweenness centrality (BC) at year 2007. (c) The cumulative distribution of BC of links added at different times. The inset reports the percentage of edges added at a certain time which are ranked in the top n positions according to the BC. Different curves correspond to $n = 100, 500, 1000$.

Evolution: two processes (Groane 1833-2007)



Two different categories of new links: 'densification' and 'exploration' clearly identified by the BC impact

Evolution of road networks: Planning vs. self-organization

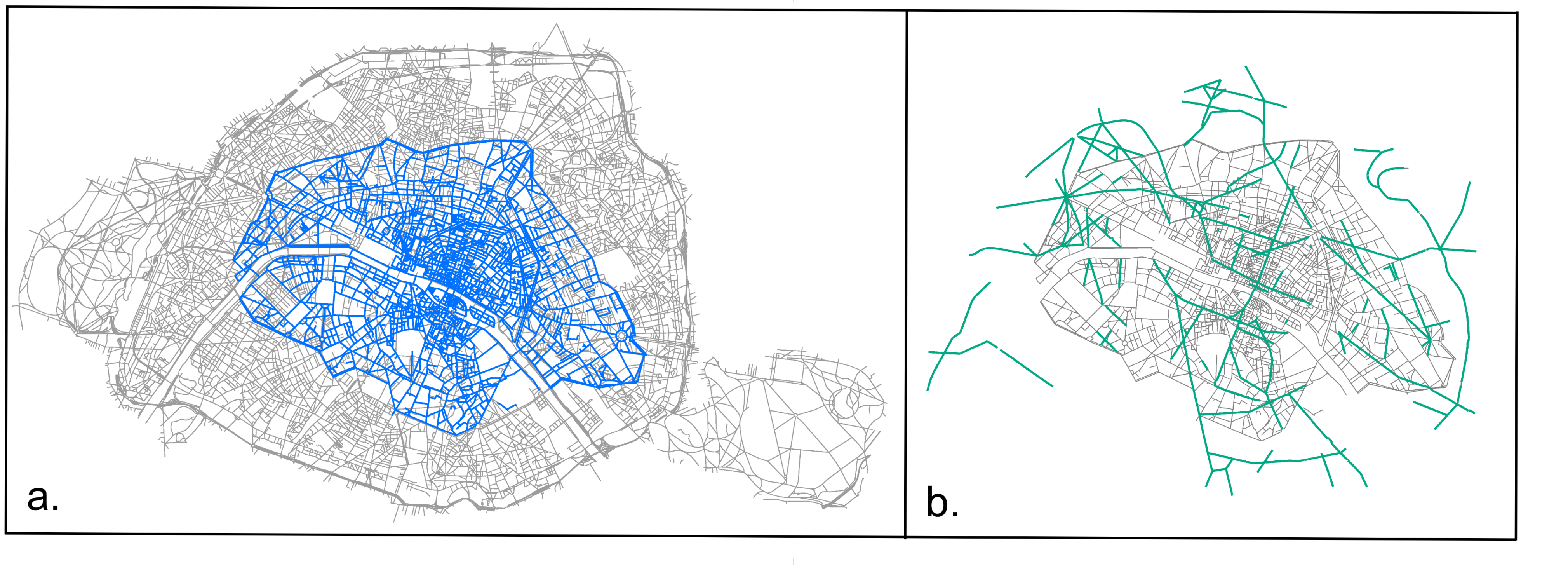
MB, Bordin, Berestycki, Gribaudi

Nature Scientific Reports, 3:2153 (2013)

Importance of planning

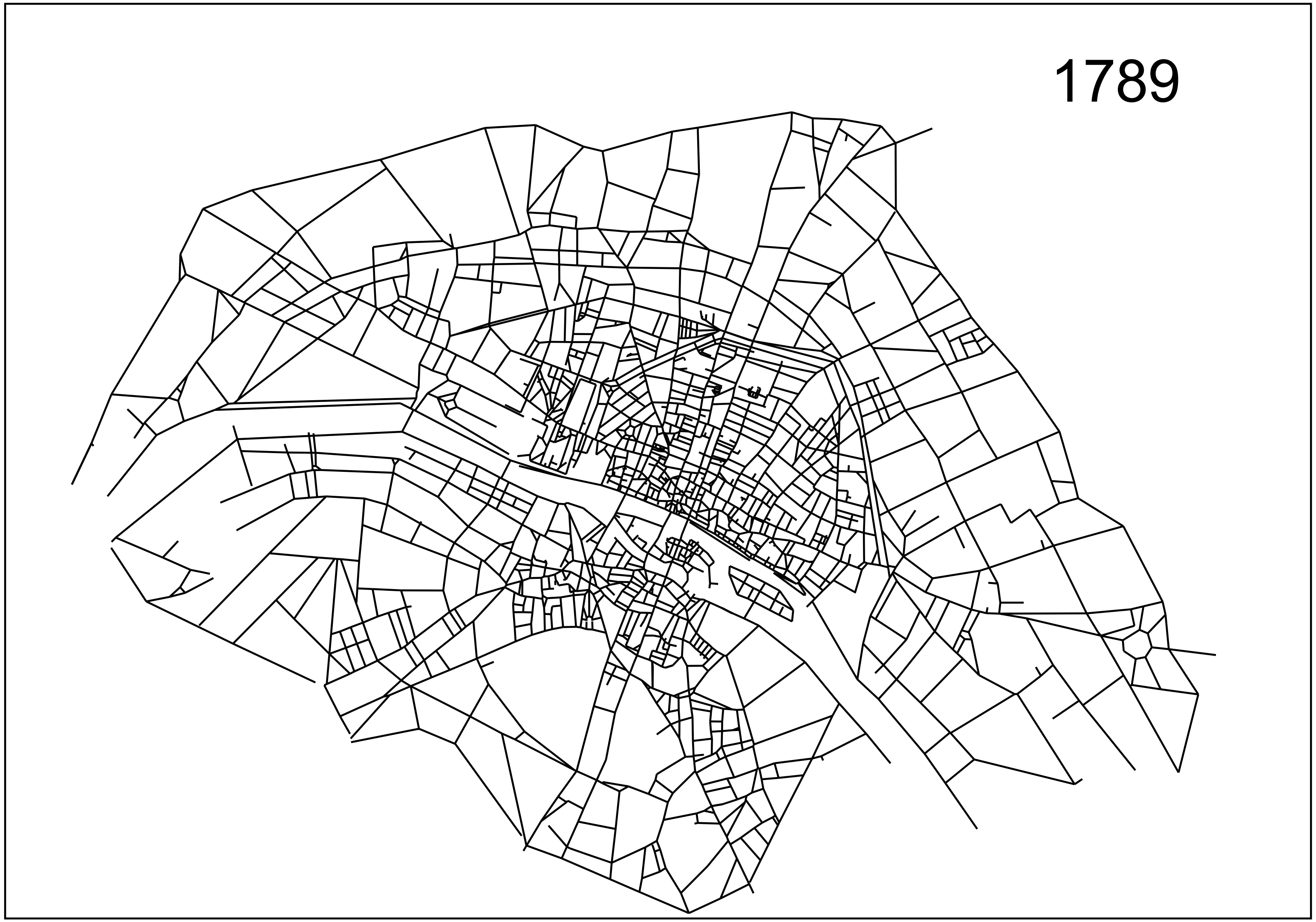
- Evolution of the Paris street network 1789-2010
- Portion defined by Paris in 1789

(1789, 1826, 1836, 1888, 1999, 2010: Verniquet, Vasserot, Poubelle)



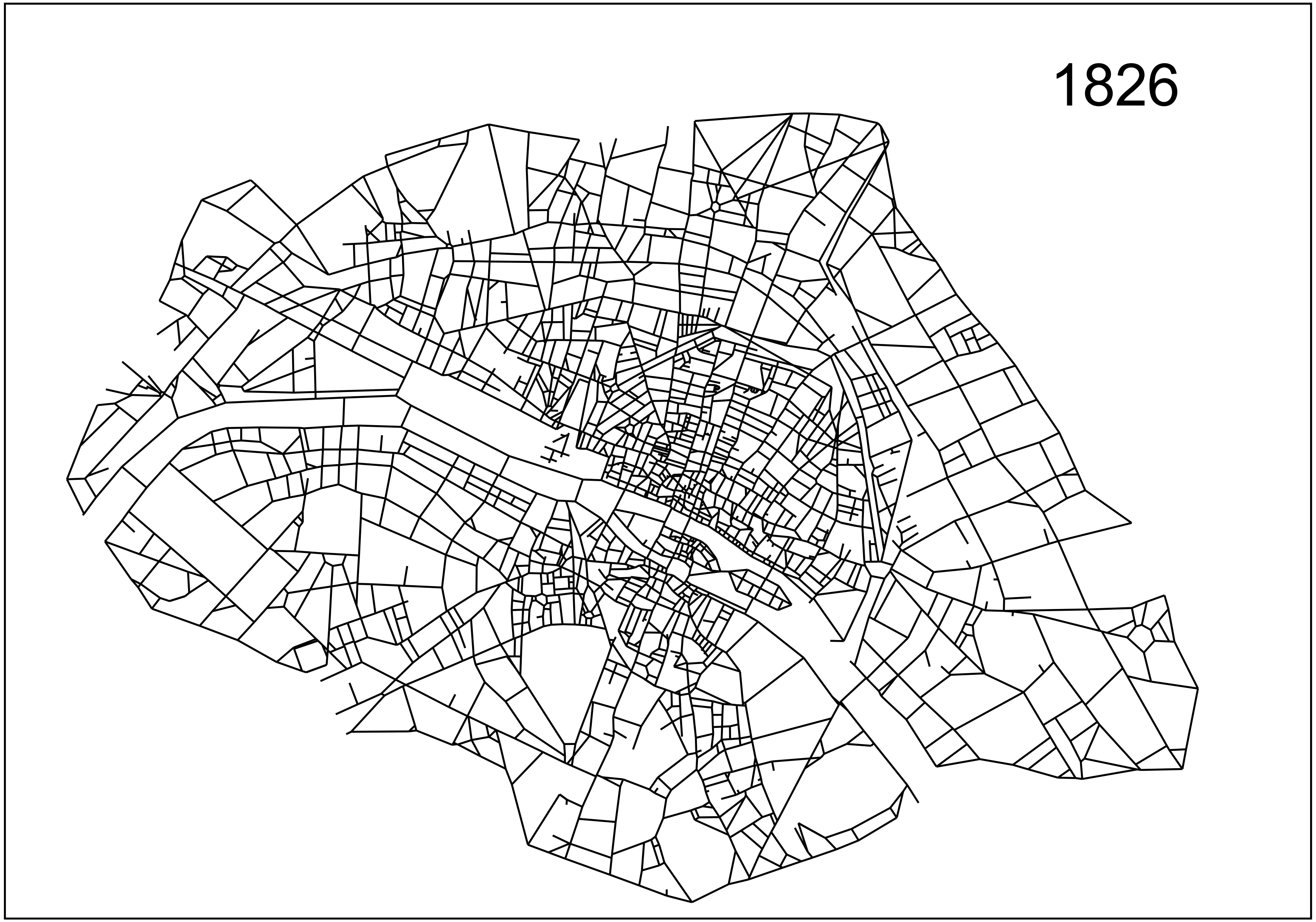
- Haussmann period (~1853-1870)

Road network evolution
Central Paris, France 1789-2010



MB, Bordin, Berestycki, Gribaudi (2013)

Road network evolution
Central Paris, France 1789-2010



MB, Bordin, Berestycki, Gribaudi (2013)

Road network evolution
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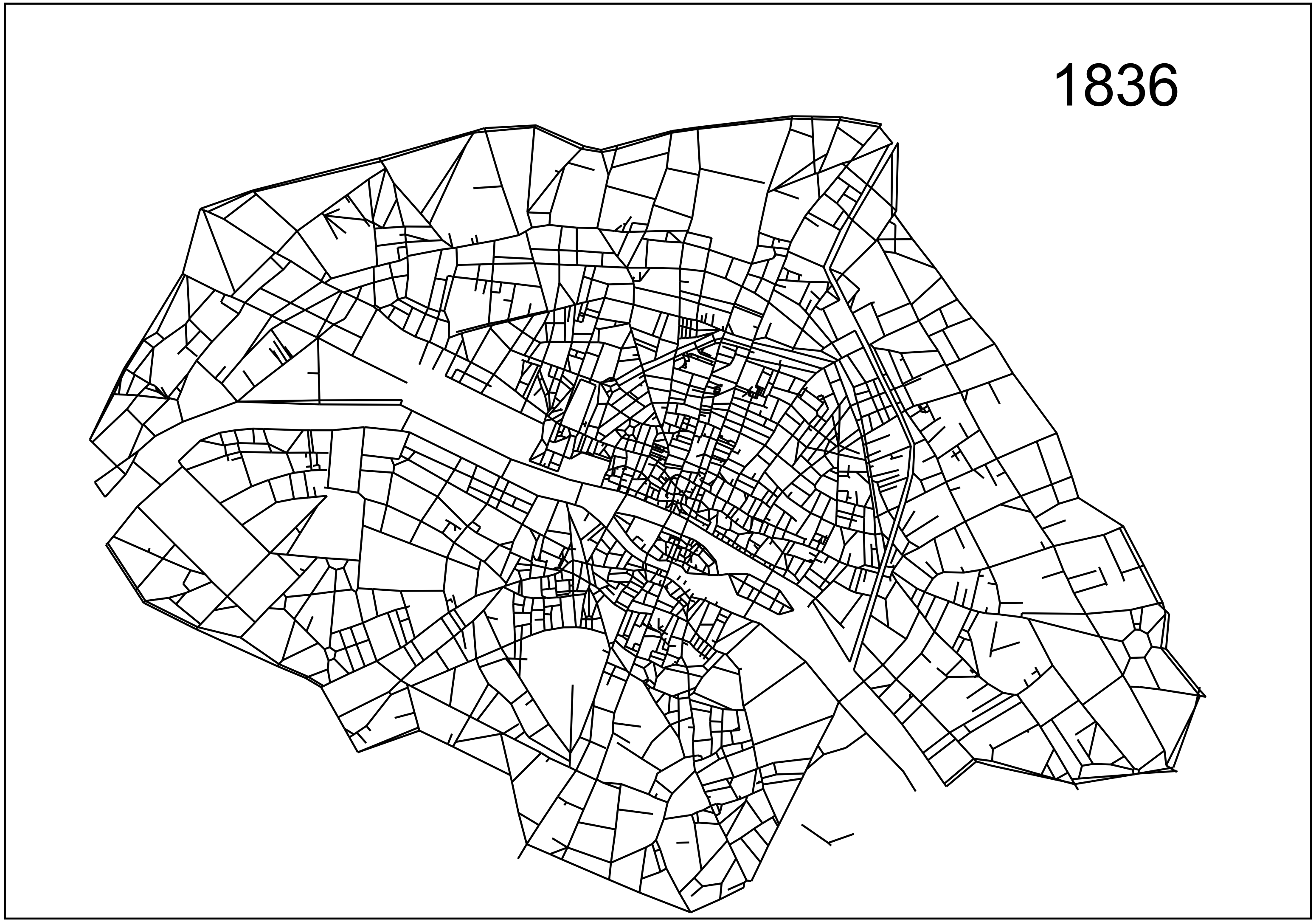
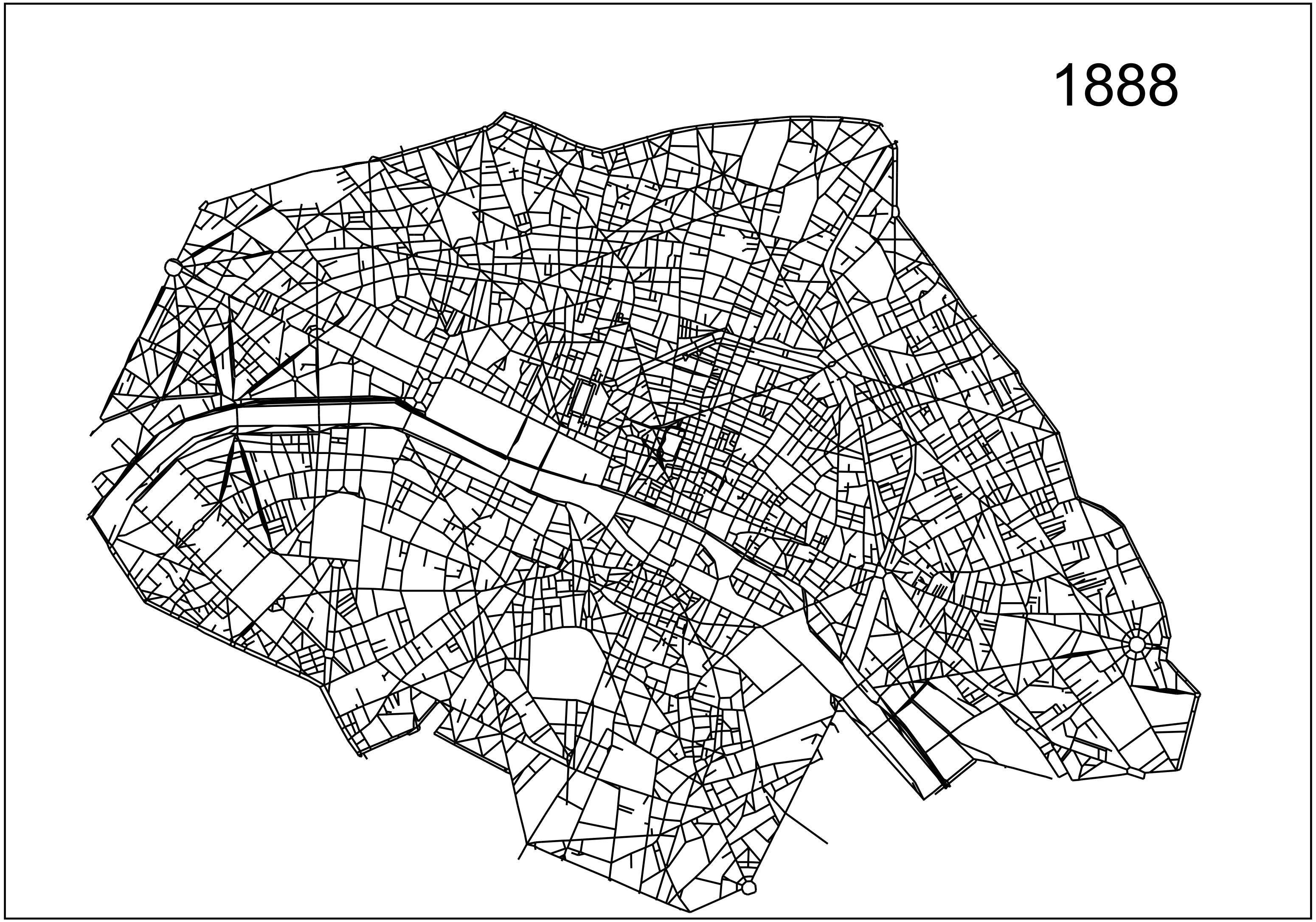


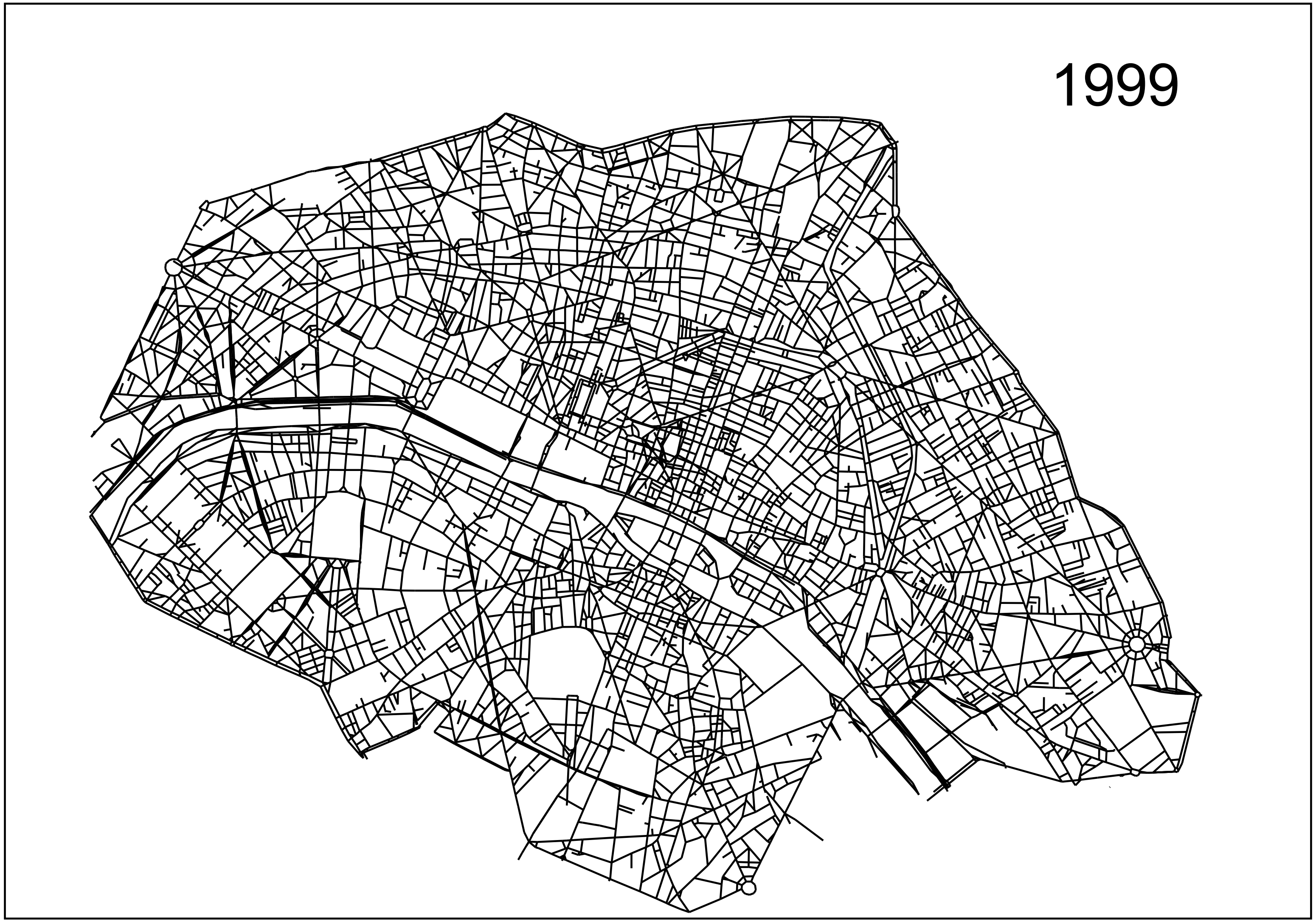
MB, Bordin, Berestycki, Gribaudi (2013)

Road network evolution
Central Paris, France 1789-2010



MB, Bordin, Berestycki, Gribaudi (2013)

Road network evolution
Central Paris, France 1789-2010

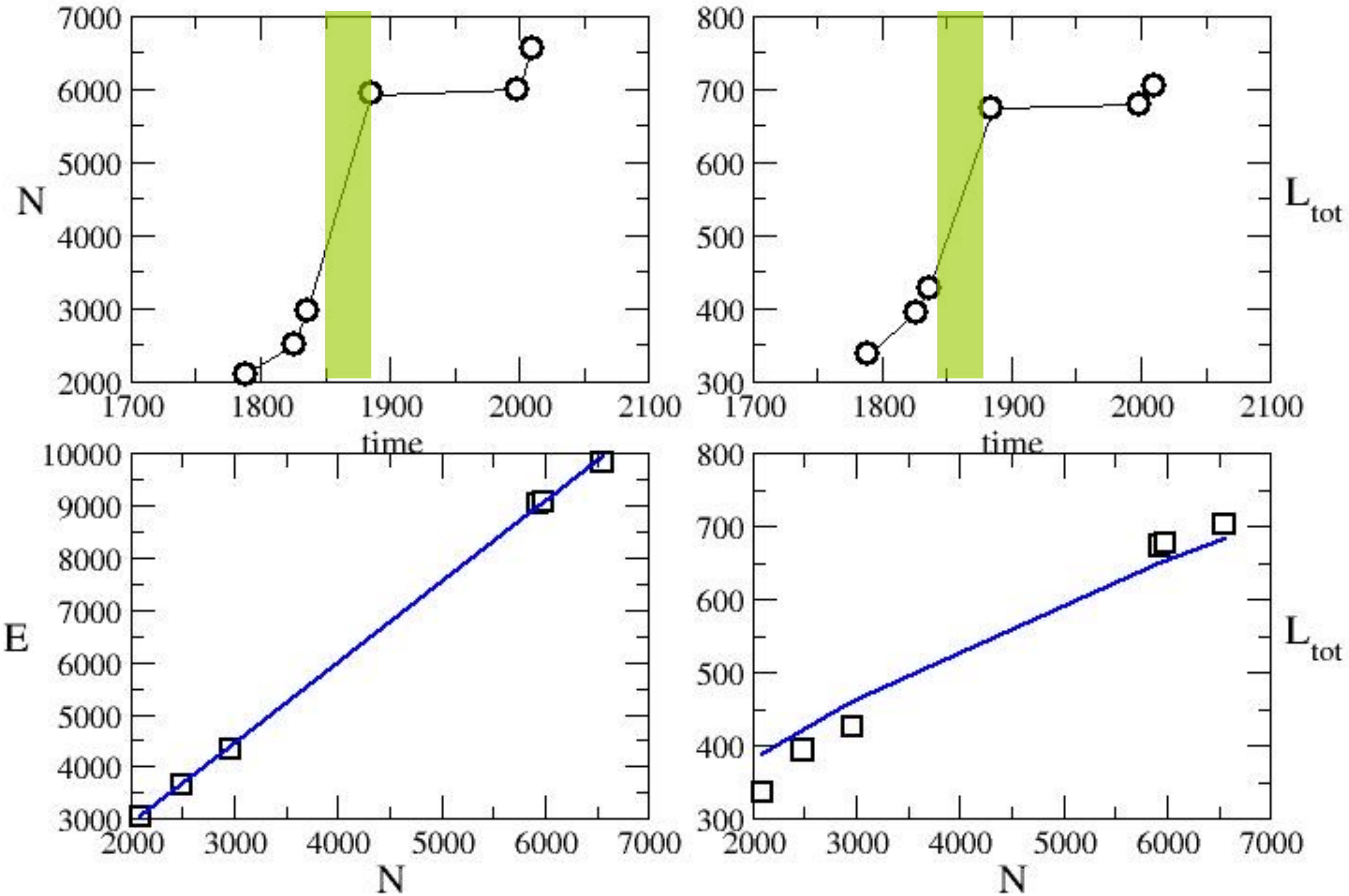


MB, Bordin, Berestycki, Gribaudi (2013)

Importance of planning

- Standard indicators versus time or N

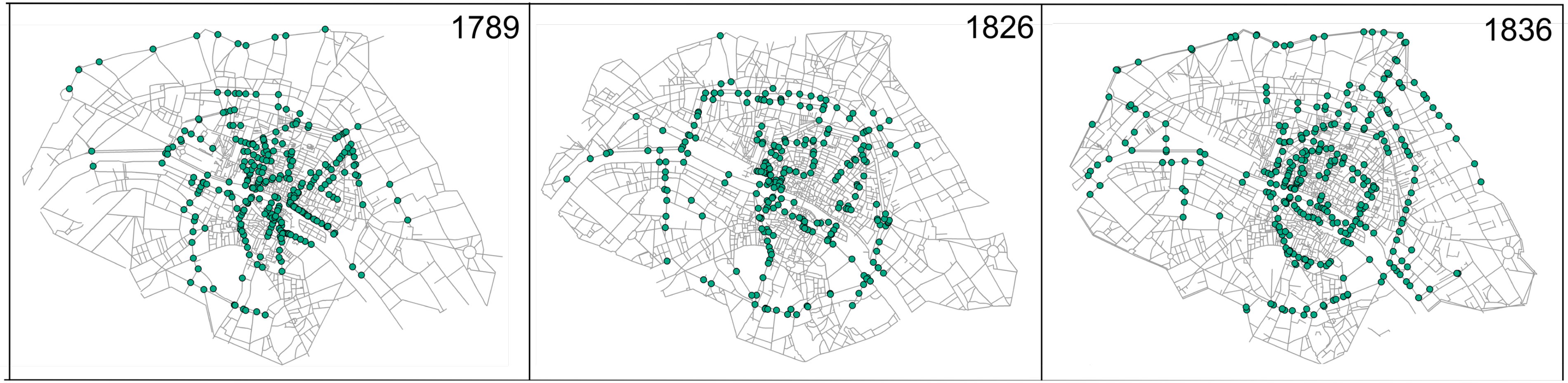
$$\frac{\Delta L_{tot}}{\Delta t} \approx 1.6 \ \text{km/year}$$



$$L_{tot} \sim \frac{\langle k \rangle}{2} \sqrt{AN}$$

Haussmann effect

- Spatial distribution of centrality (most central nodes)



Haussmann effect

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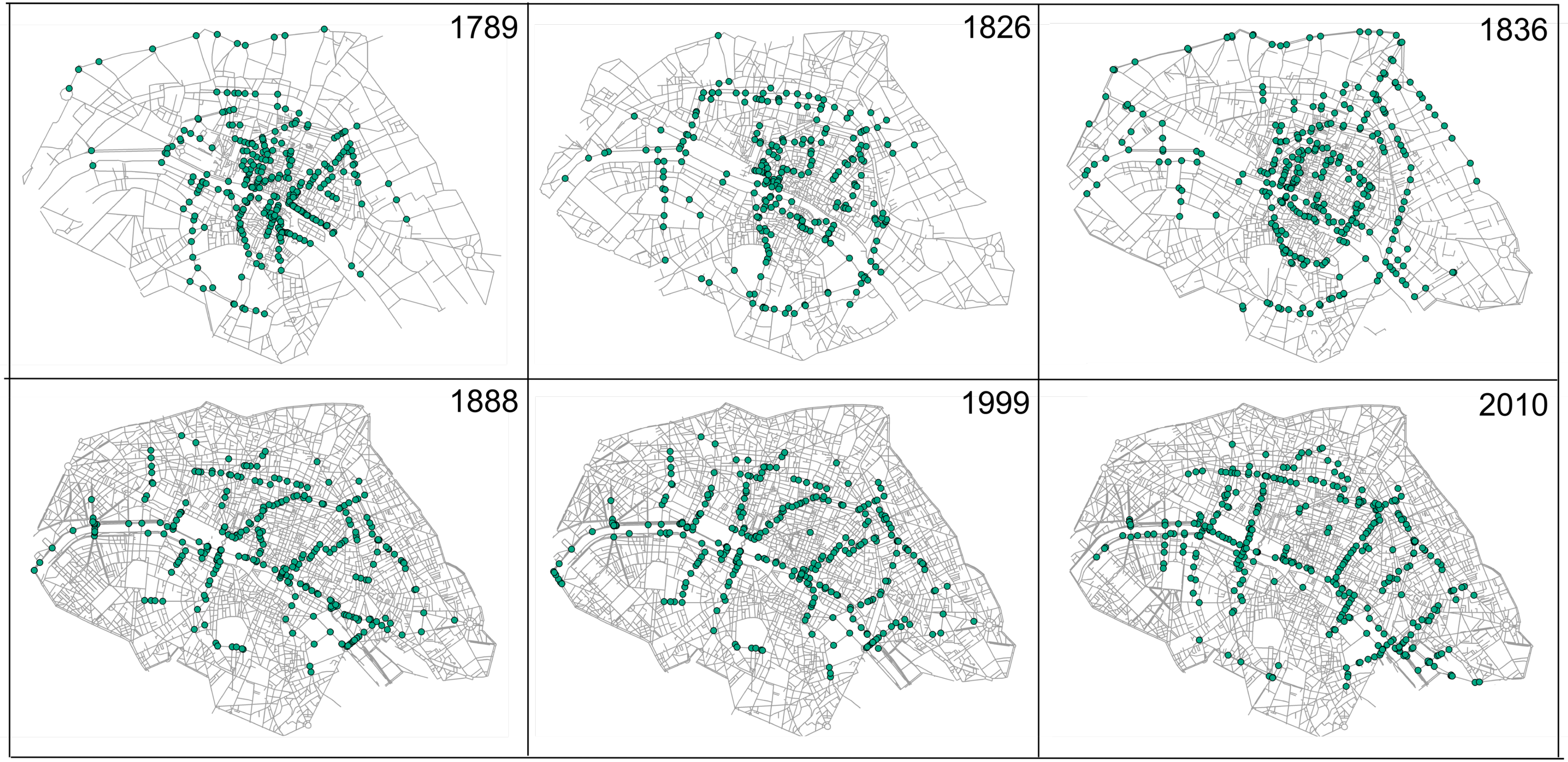


Short recap

- New results on new datasets usually imply to have new tools
- Natural 'organic' evolution…
  - Two main processes: densification vs. exploration
  - long duration, small spatial scale
  - Interesting stylized facts (centrality, area and form factor distribution)
- … vs. planning
  - Signature: spatial distribution of centrality (modification of paths) and change of the block shape distribution ('non-natural')
  - Does not respect the existing geometry: short duration but large spatial scales

Other structures: railways and subways

Subway evolution: Not a new problem

- Cope (1967): Stages of the London underground rail

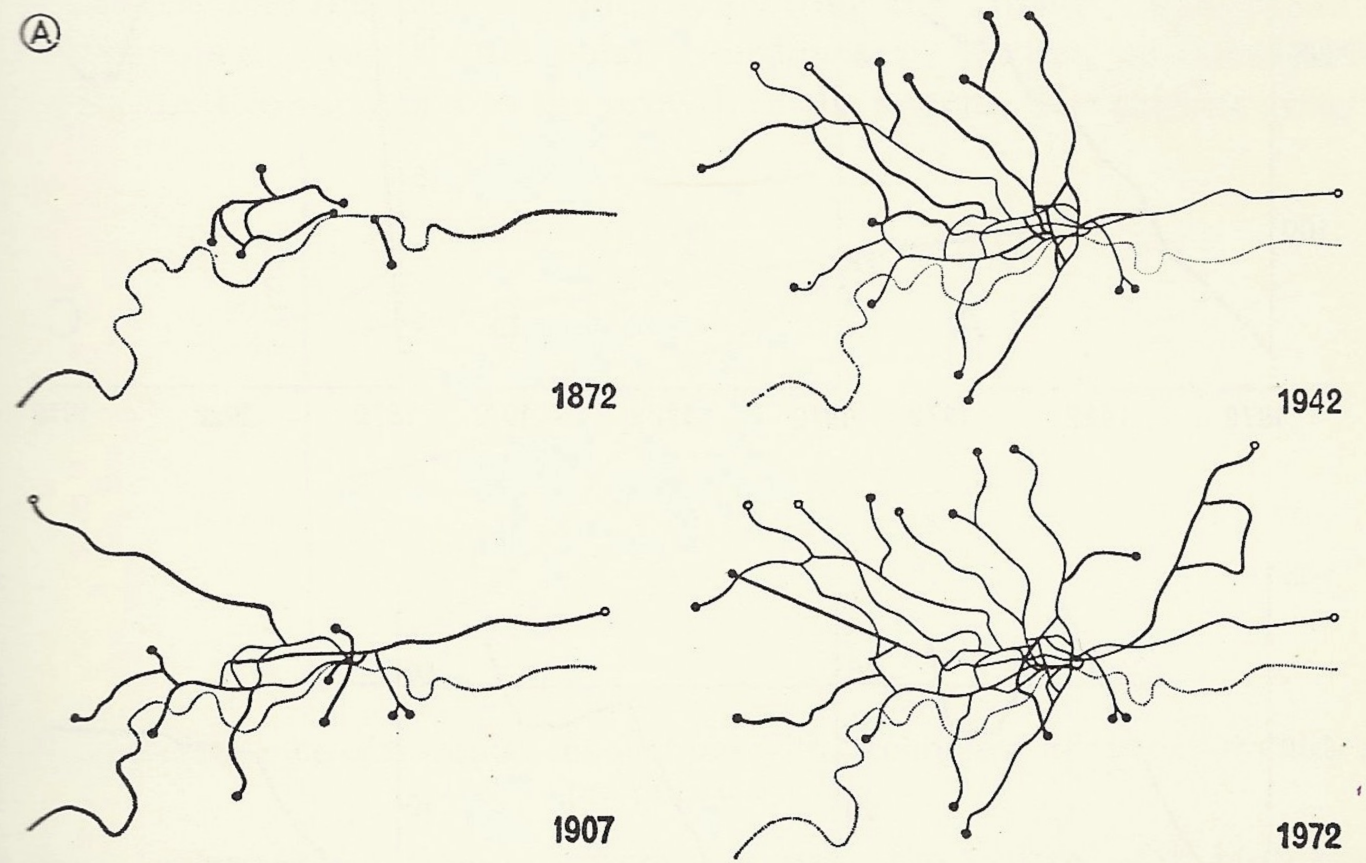
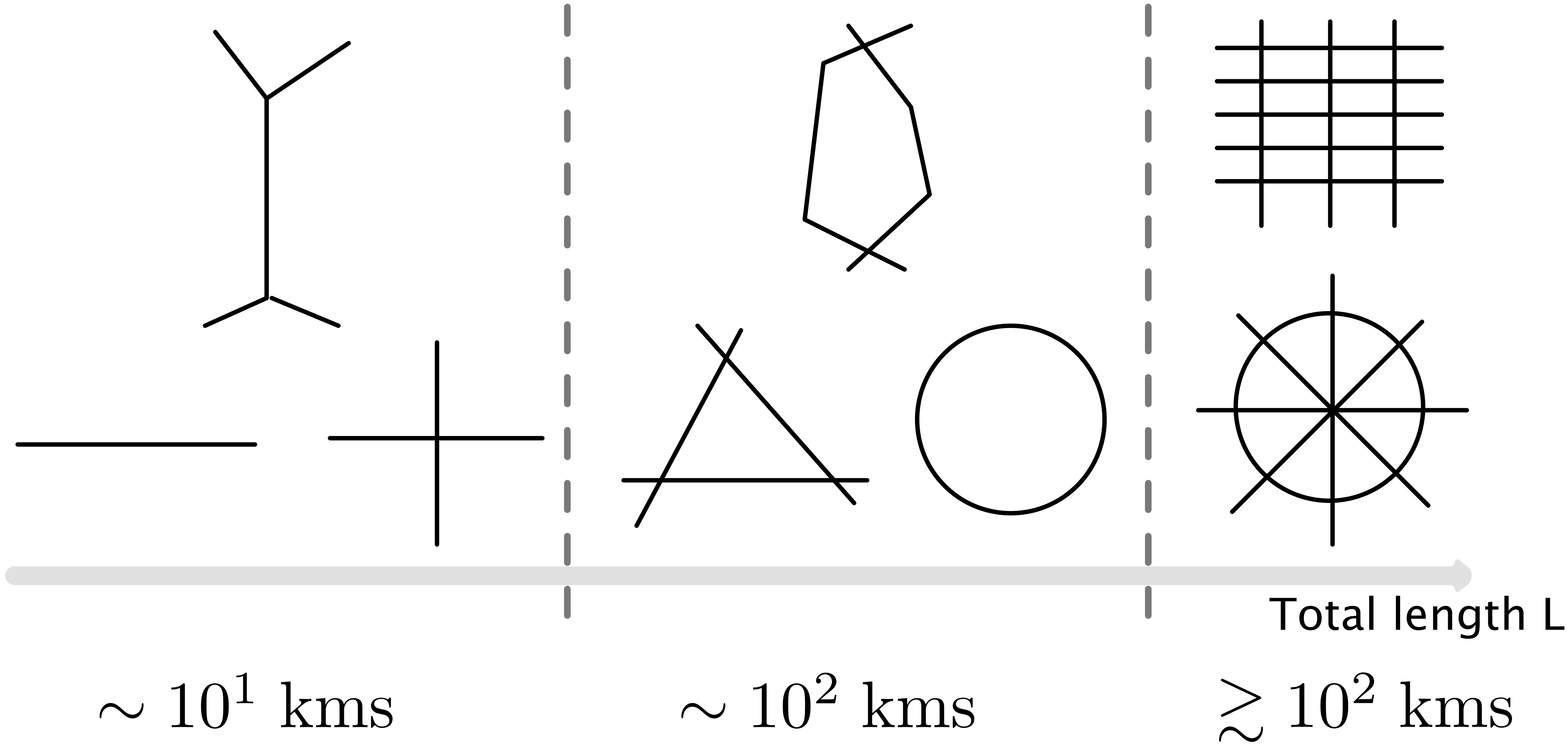


Fig. 5.7. Stages in the growth of the London underground rail network. Source: Cope, 1967.

Evolution of the subway structure



- For small L we observe a line or a simple tree.
- For larger L we observe the appearance of a loop
- For much larger L more complex shapes including a lattice like network or a superimposition of a ring and radial lines.

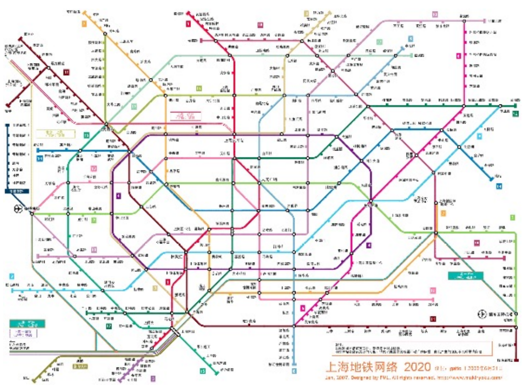
World subway networks

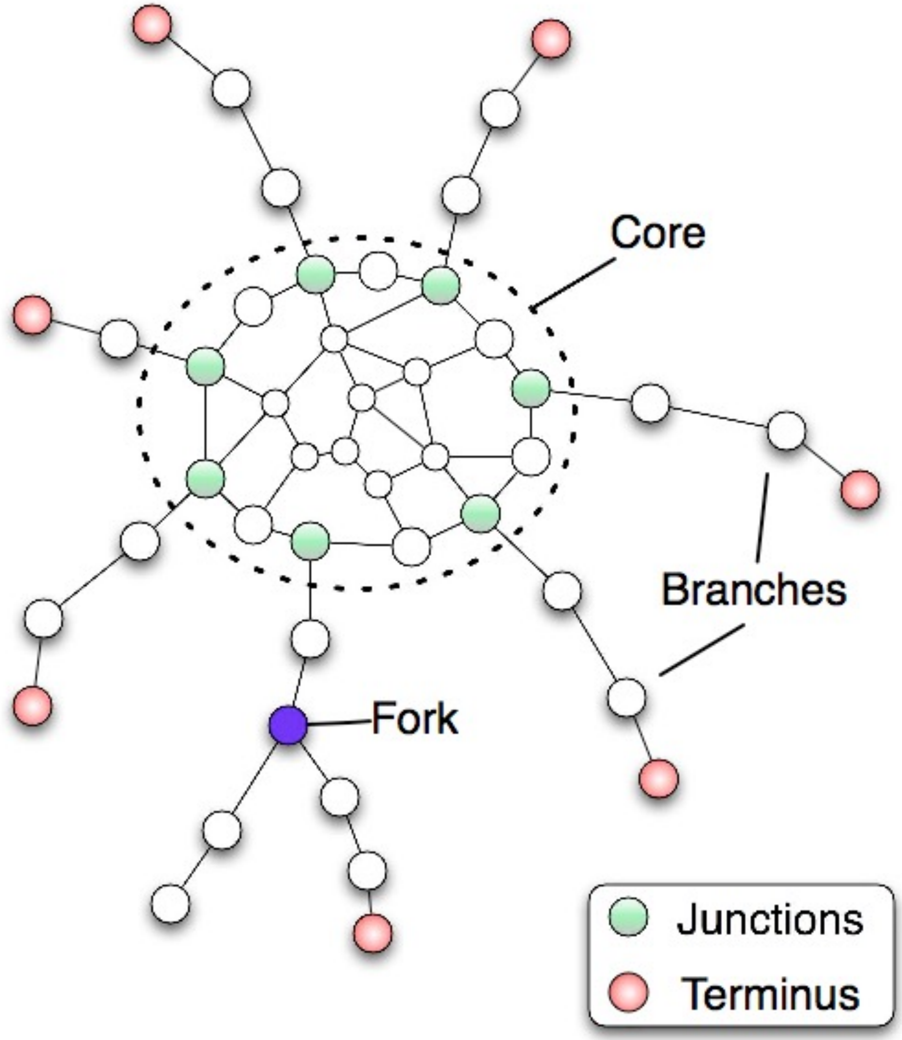
We focus on large networks (N>100 stations)

city	P (millions)	N_L	N	$\overline{\ell_1}$ (km)	ℓ_T (km)	ℓ_T/ℓ_T^{reg}	β (%)
Beijing	19.6	9	104	1.79	204	0.14	39
Tokyo	12.6	13	217	1.06	279	0.13	43
Seoul	10.5	9	392	1.39	609	0.39	38
Paris	9.6	16	299	0.57	205	0.18	38
Mexico City	8.8	11	147	1.04	170	0.15	39
NYC	8.4	24	433	0.78	373	0.12	36
Chicago	8.3	11	141	1.18	176	0.08	71
London	8.2	11	266	1.29	397	0.20	47
Shanghai	6.9	11	148	1.47	233	0.21	61
Moscow	5.5	12	134	1.67	260	0.16	71
Berlin	3.4	10	170	0.77	141	0.30	60
Madrid	3.2	13	209	0.90	215	0.42	46
Osaka	2.6	9	108	1.12	137	0.88	43
Barcelona	1.6	11	128	0.72	103	0.32	38

$$\ell_T^{reg} \sim \frac{\langle k \rangle}{2}\sqrt{AN}$$

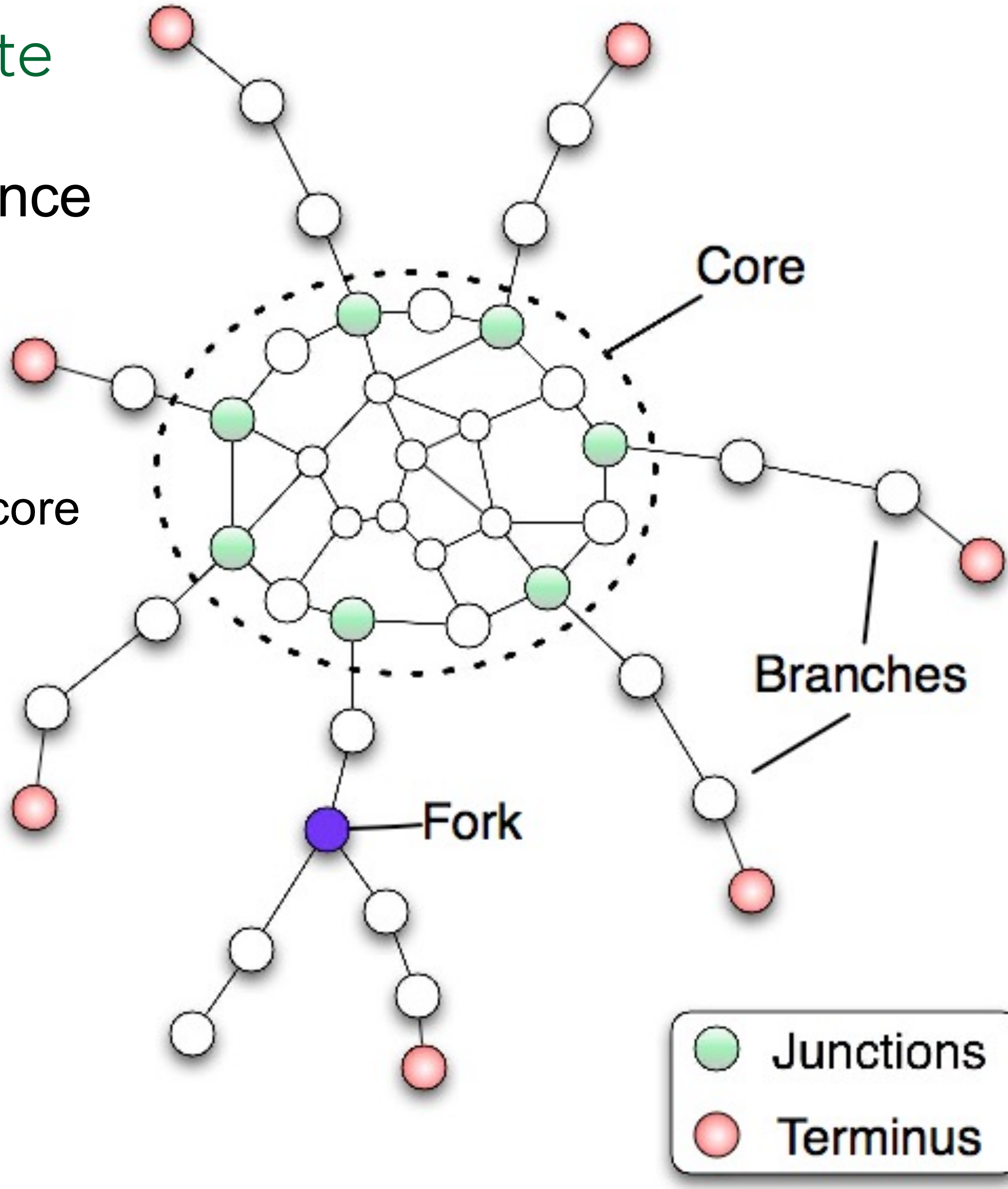
"Universal" template





"Universal" template

- Quantitative convergence
  - Fraction of branches stations of order 50%
  - Extension of branches/core extension of order 2
  - Average degree of core of order 2.5 and f_2>60%



Digression: destruction/reconstruction

Sometimes the data is not enough for a quantitative description…

Le Havre (before bombing WWII -1944)

Bertin, Galli, Rossi, J. Urban Design 2022

Le Havre (after bombing WWII -1944)

Bertin, Galli, Rossi, J. Urban Design 2022

Le Havre (after reconstruction)

Bertin, Galli, Rossi, J. Urban Design 2022

Beyond infrastructures: economic activity

with Julie Gravier

Beyond road networks: economic activity

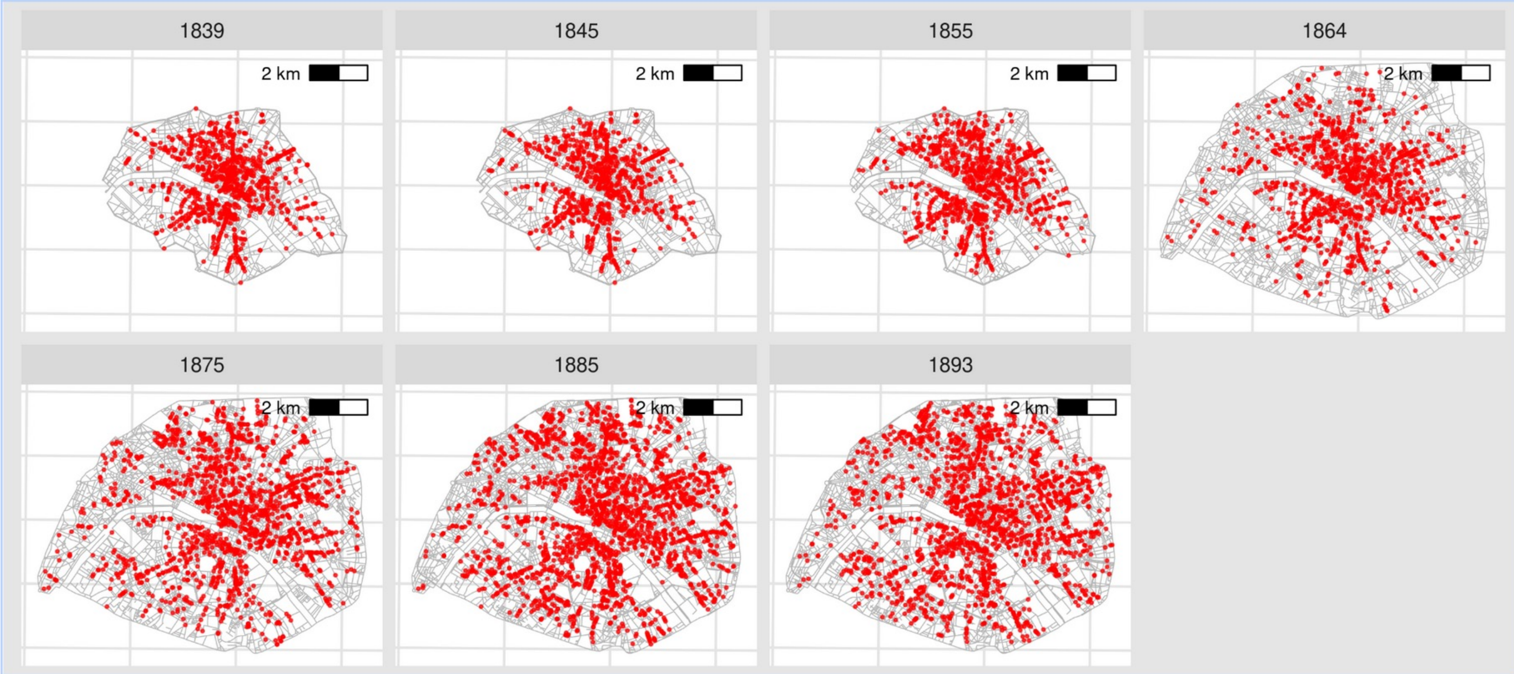
- Spatial distribution of activities: a standard topic in spatial economics

  - Evolution of activities (number, type, …)

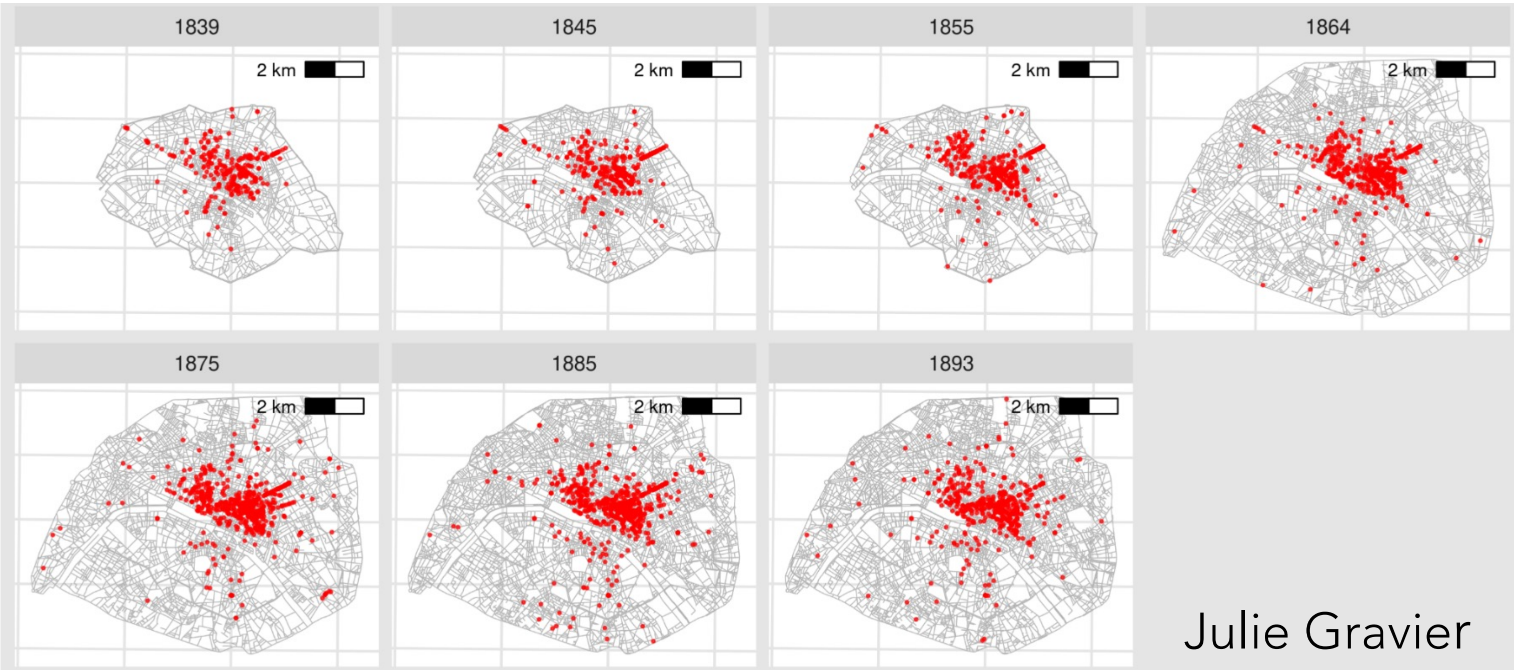
  - Spatial distribution of activities (interactions ?)

Different activities: spatial distribution and evolution ?

Grocery stores



Jewelry



Julie Gravier

Beyond road networks: economic activity

- Opportunity to bring a historical light on this problem ! Important step in the quantitative study of the evolution of cities…

- Future: other aspects of life in cities with the help of new data sources…

Thank you for your attention.

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Additional slides

An old problem in quantitative geography

- Kansky (63-69)
Evolution
of the Sicilian
railroad network

- Morrill (1965)
Railway network
Growth

- New data sources:
Digitization of
old maps

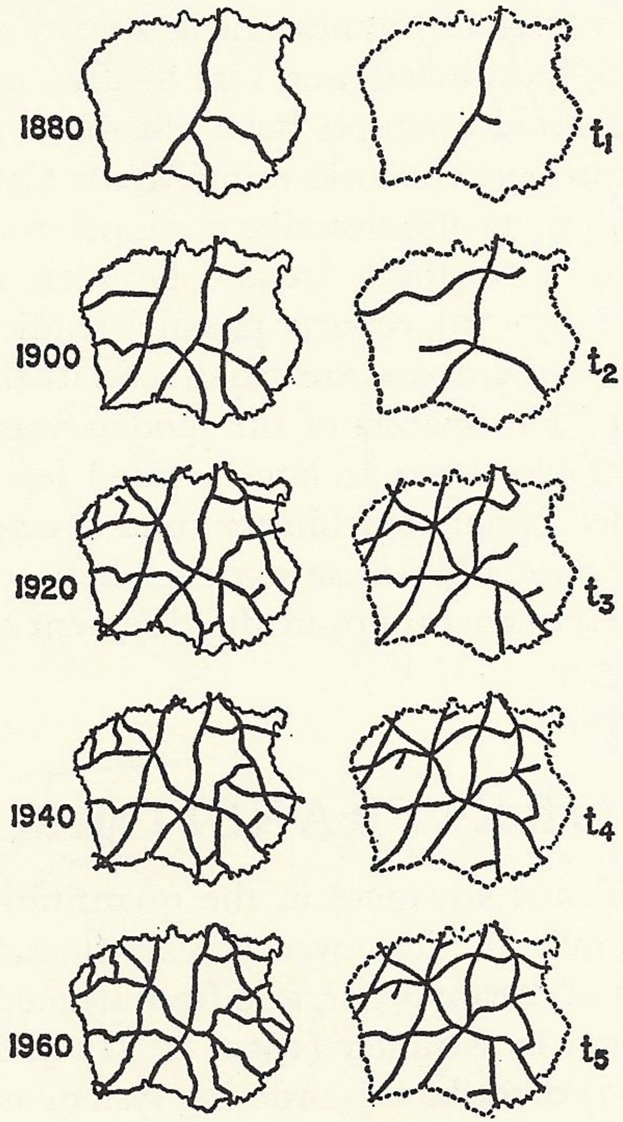
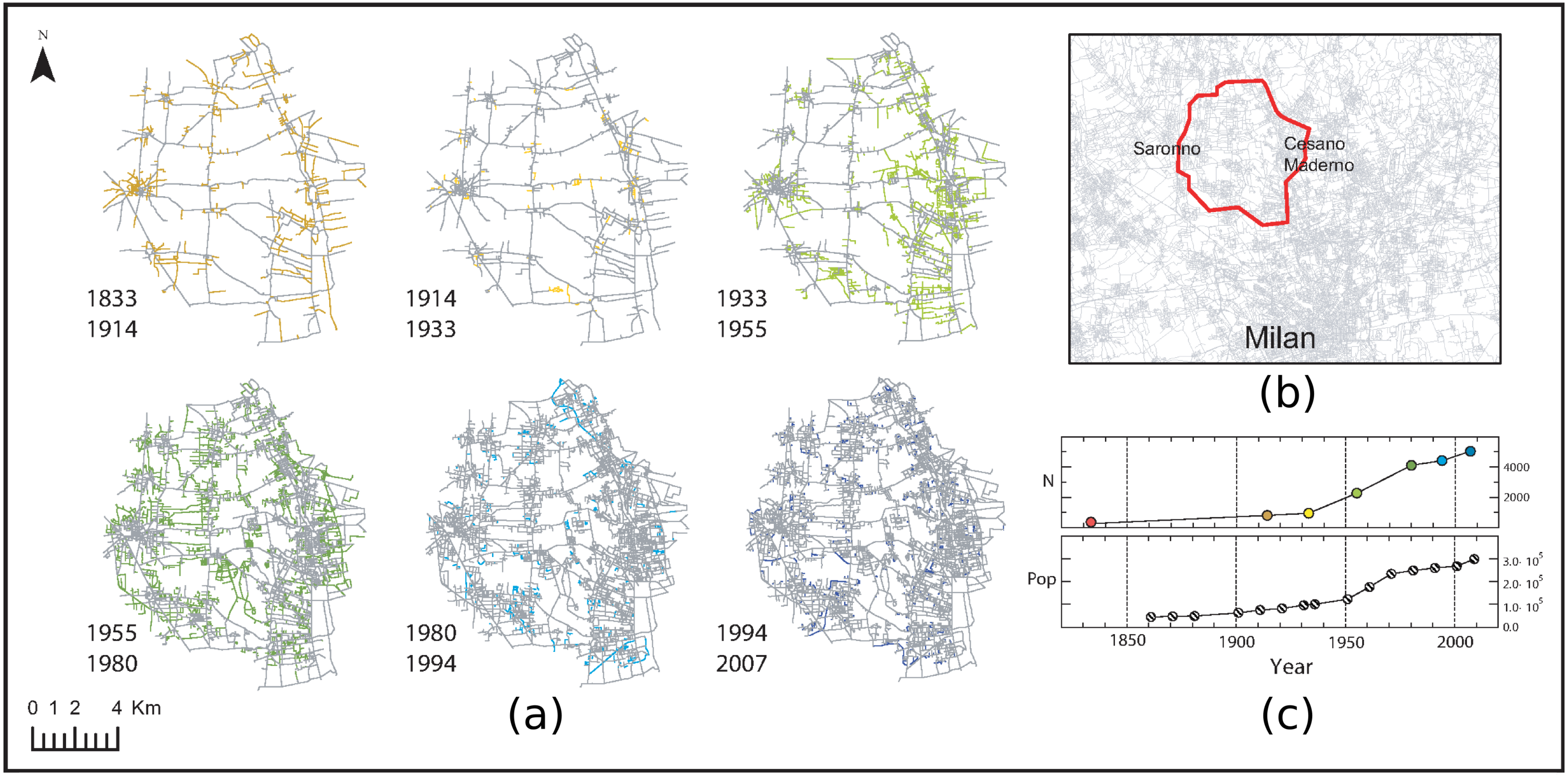


Fig. 5.30. Comparison of the (*A*) actual growth of railway network in central Sweden with (*B*) the simulated growth. Source: Morrill, 1965, pp. 130–70.

Road network evolution
Groane region, Italy 1833-2007



Strano, Nicosia, Latora, Porta, MB, Nature Scientific Reports (2012)

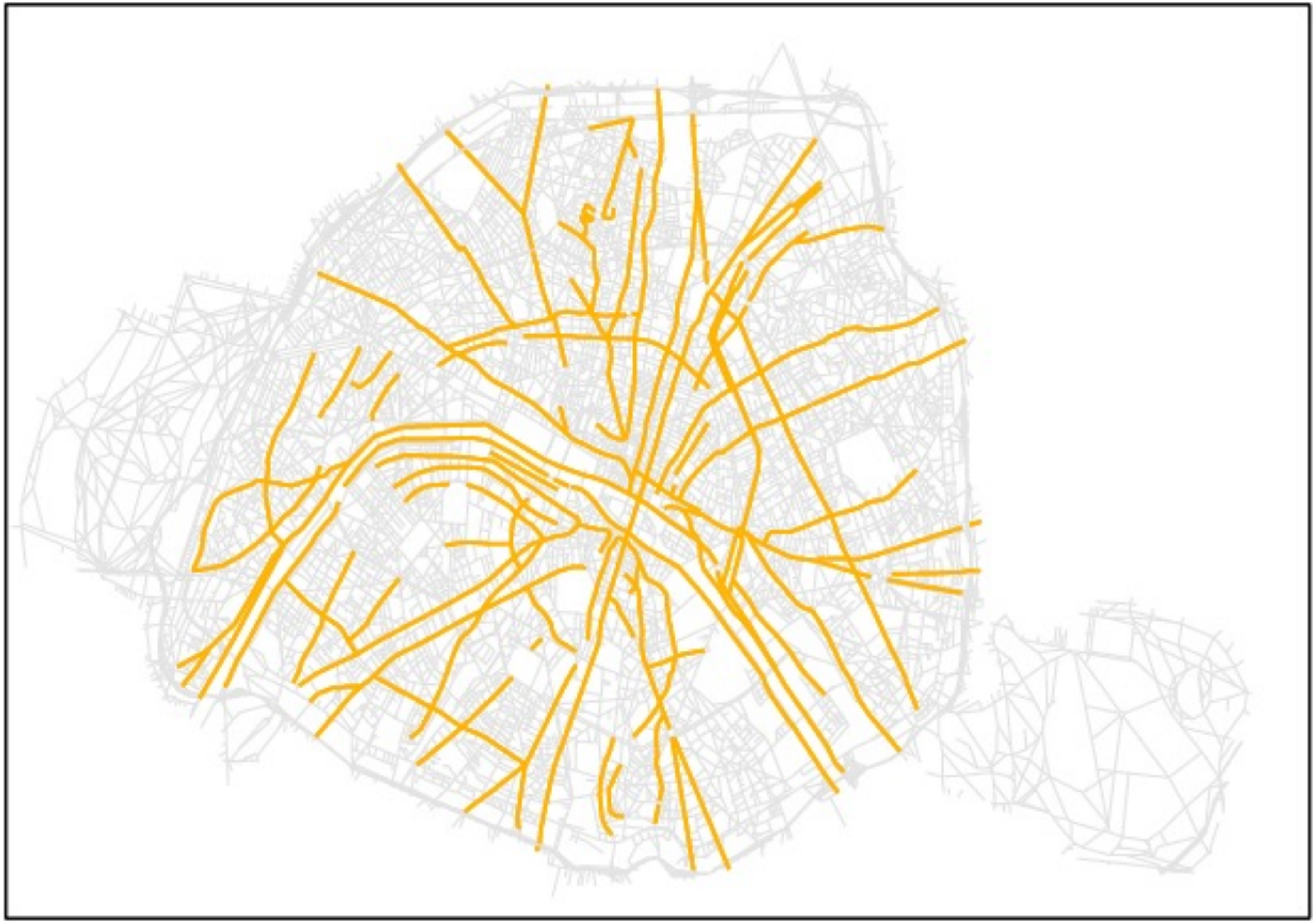
Characterization of links

- Traditional division: gallo-roman, natural, Haussmann

Gallo-roman (2% of 1999's length)

Characterization of links

- Traditional division: gallo-roman, natural, Haussmann



Natural (10% of 1999's length)

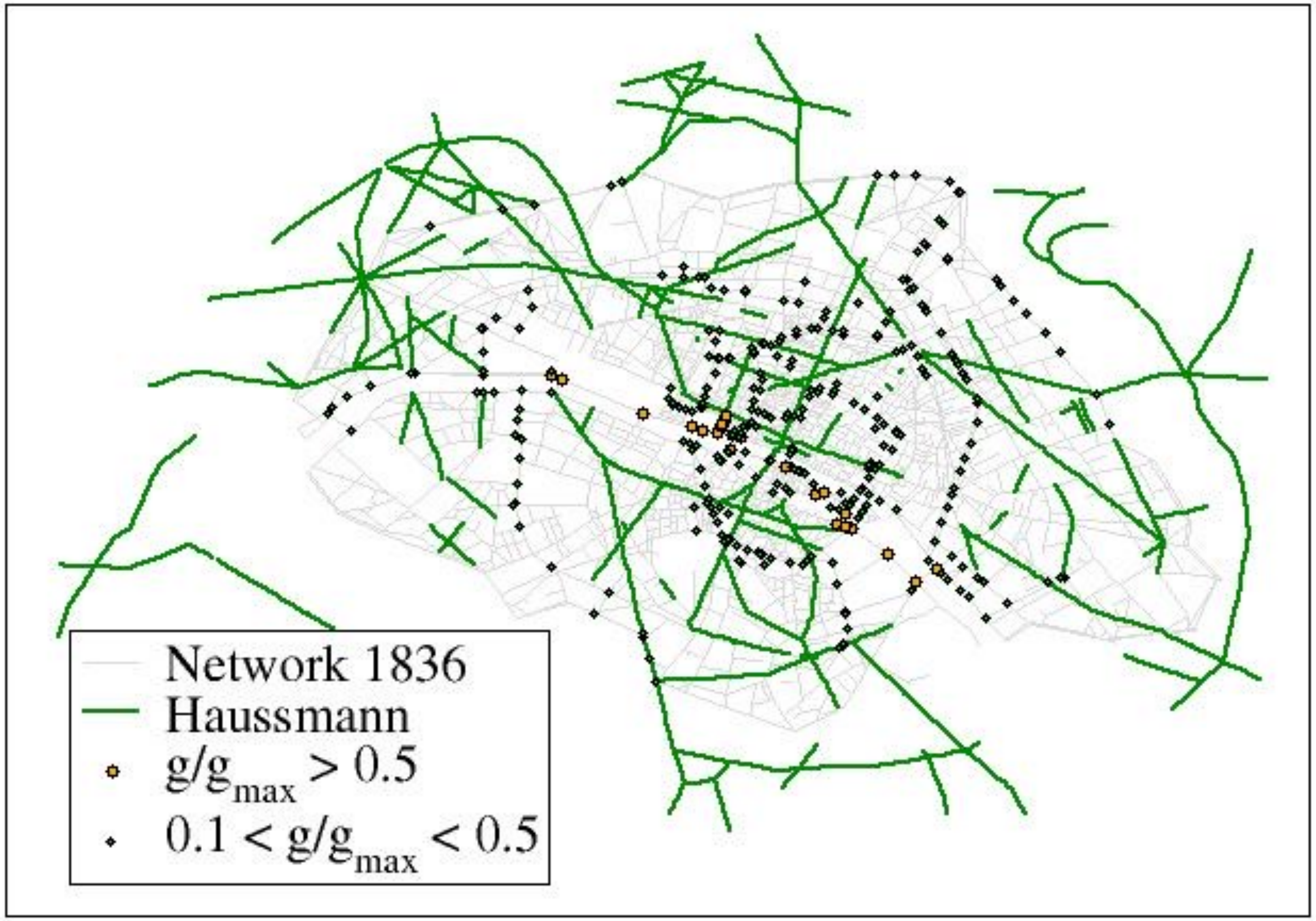
Characterization of links

- Traditional division: gallo-roman, natural, Haussmann

Haussmann (6% of 1999's length)

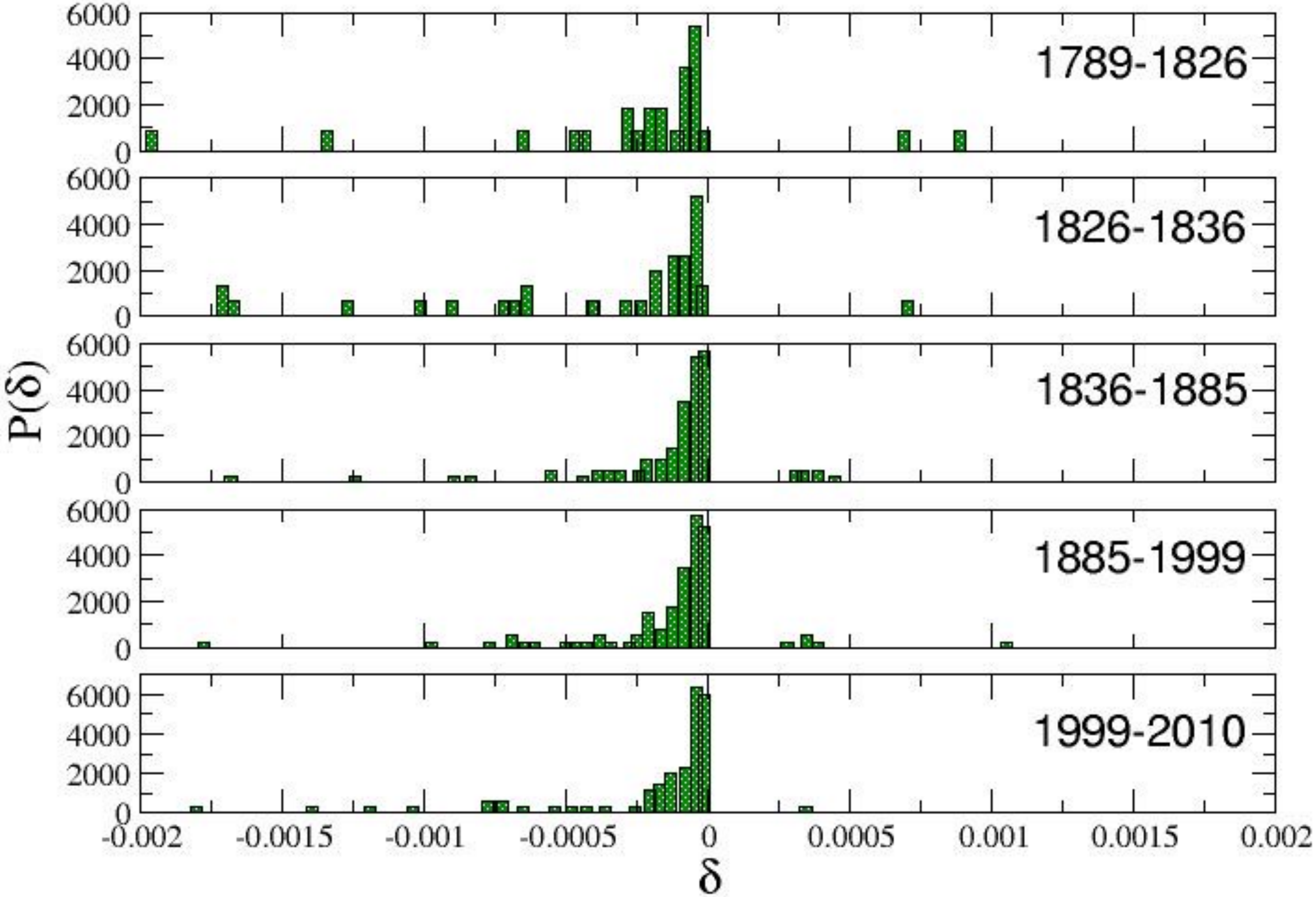
Haussmann effect

- Haussmann optimal ?



Properties of new links

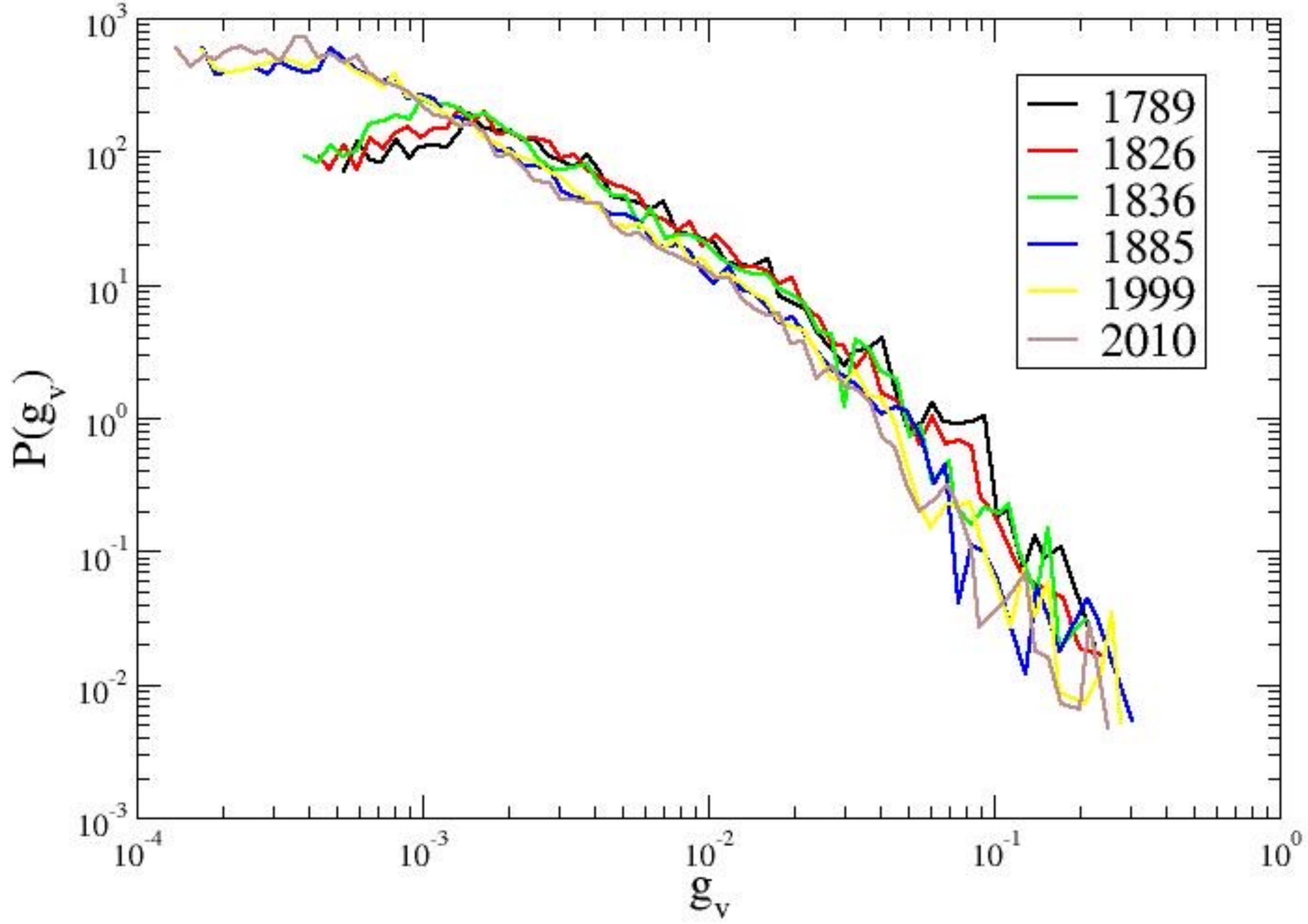
- Betweenness centrality impact



Densification is the main process here

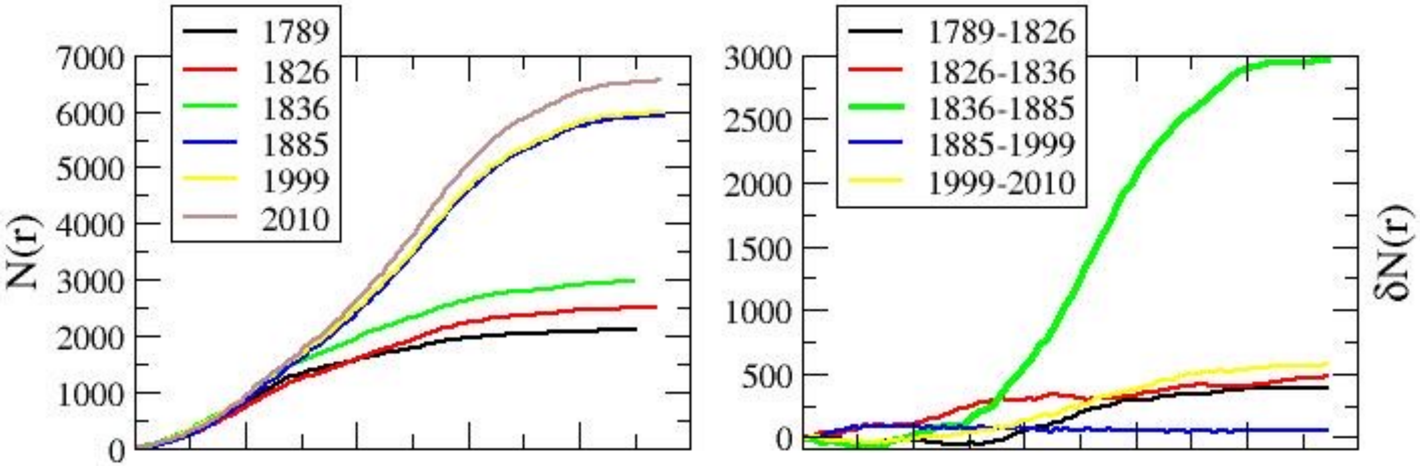
Evolution of centrality

- Stable distribution of centrality (small decrease of <g>)

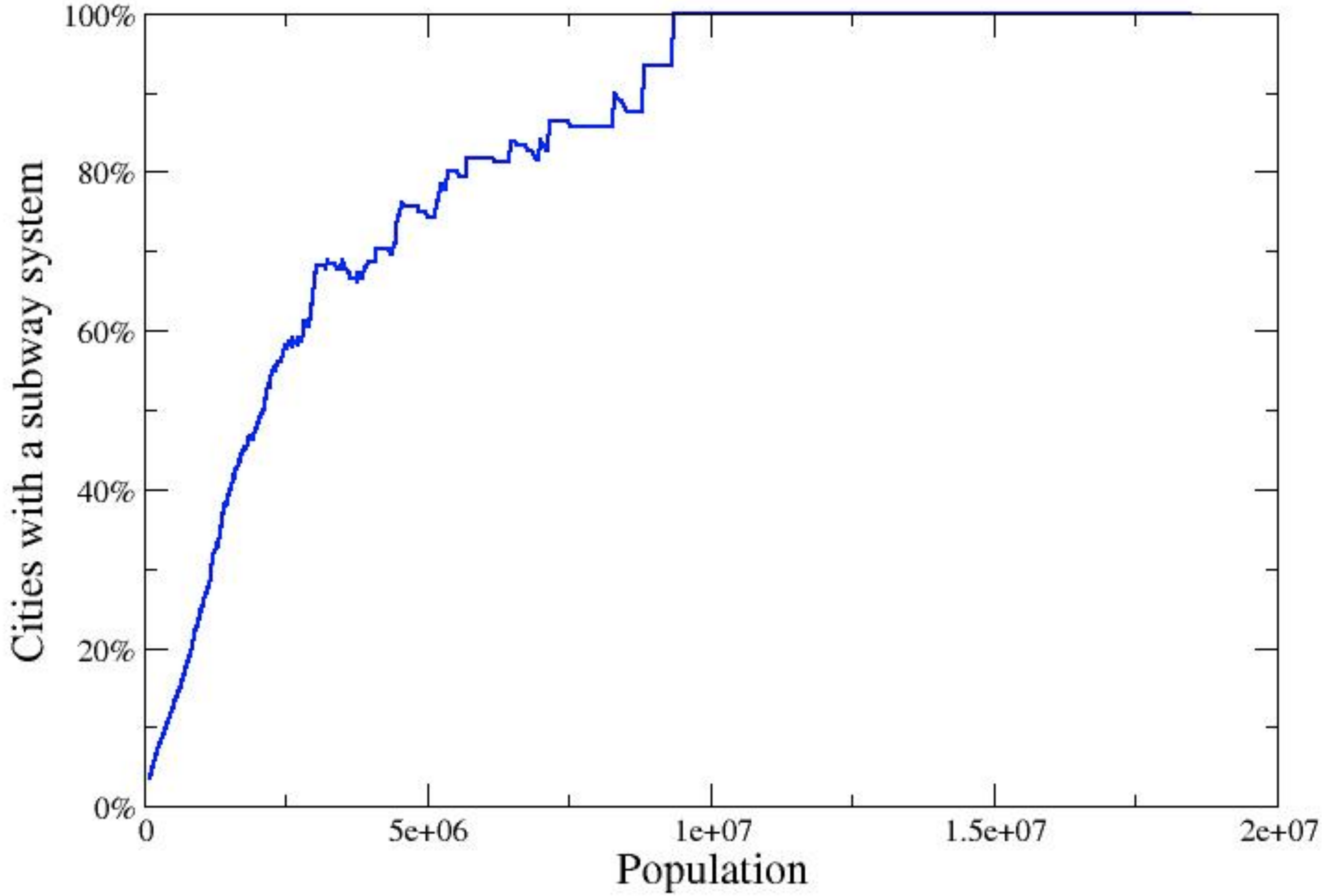


Haussmann effect

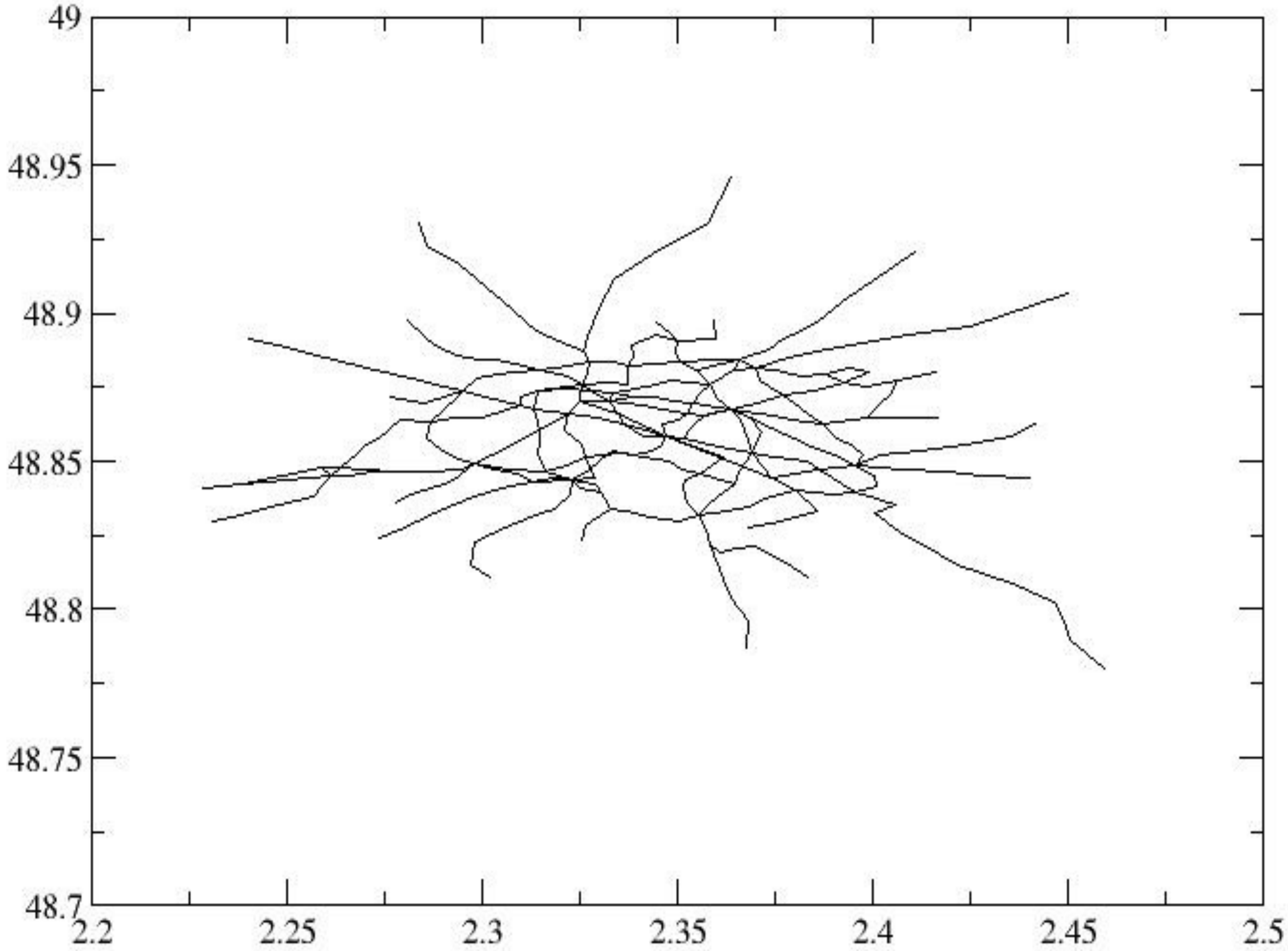
- Spatial distribution of centrality
- r: distance to barycenter



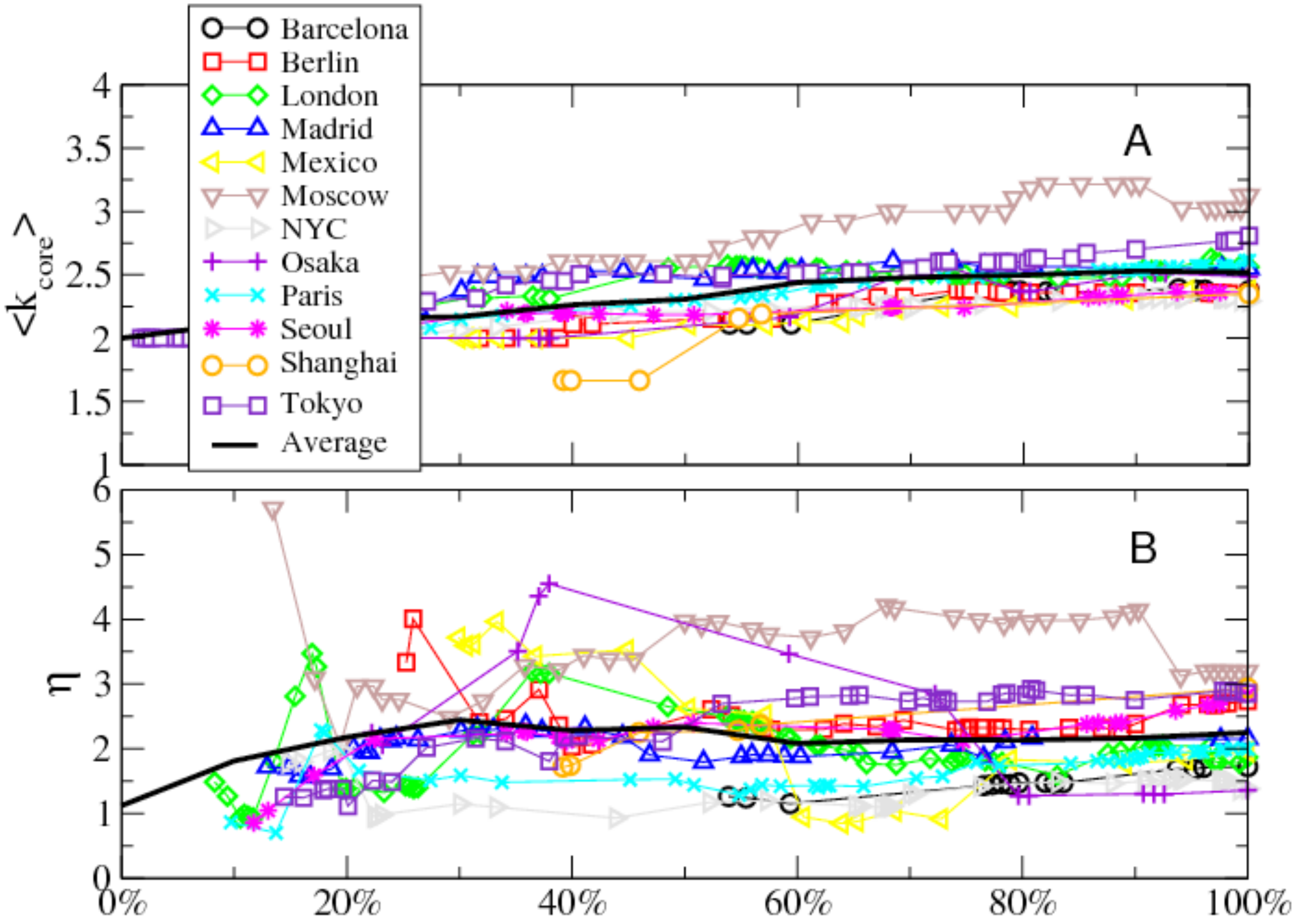
All large cities have a subway system



Spatial representation (Paris)



Evolution



Barcelona
Berlin
London
Madrid
Mexico
Moscow
NYC
Osaka
Paris
Seoul
Shanghai
Tokyo
Average
A
B
<k_core>
η
0%
20%
40%
60%
80%
100%

Evolution fraction of branches stations

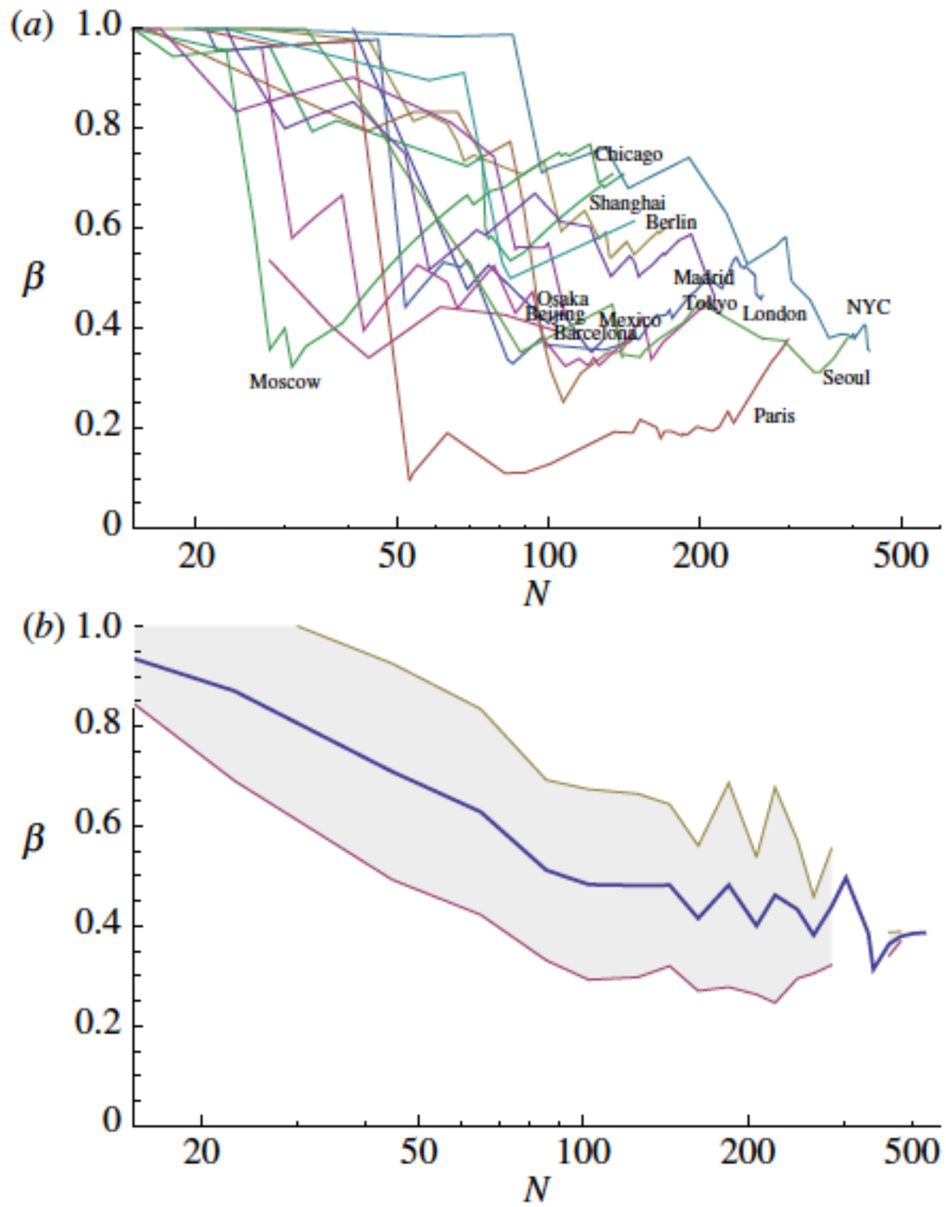


Figure 6. (*a*) Parameter β as a function of the number of stations N for the different world subways. (*b*) Same as (*a*) but averaged over 20 bins and showing the standard deviation. (Online version in colour.)

Average degree
Percentage f_2

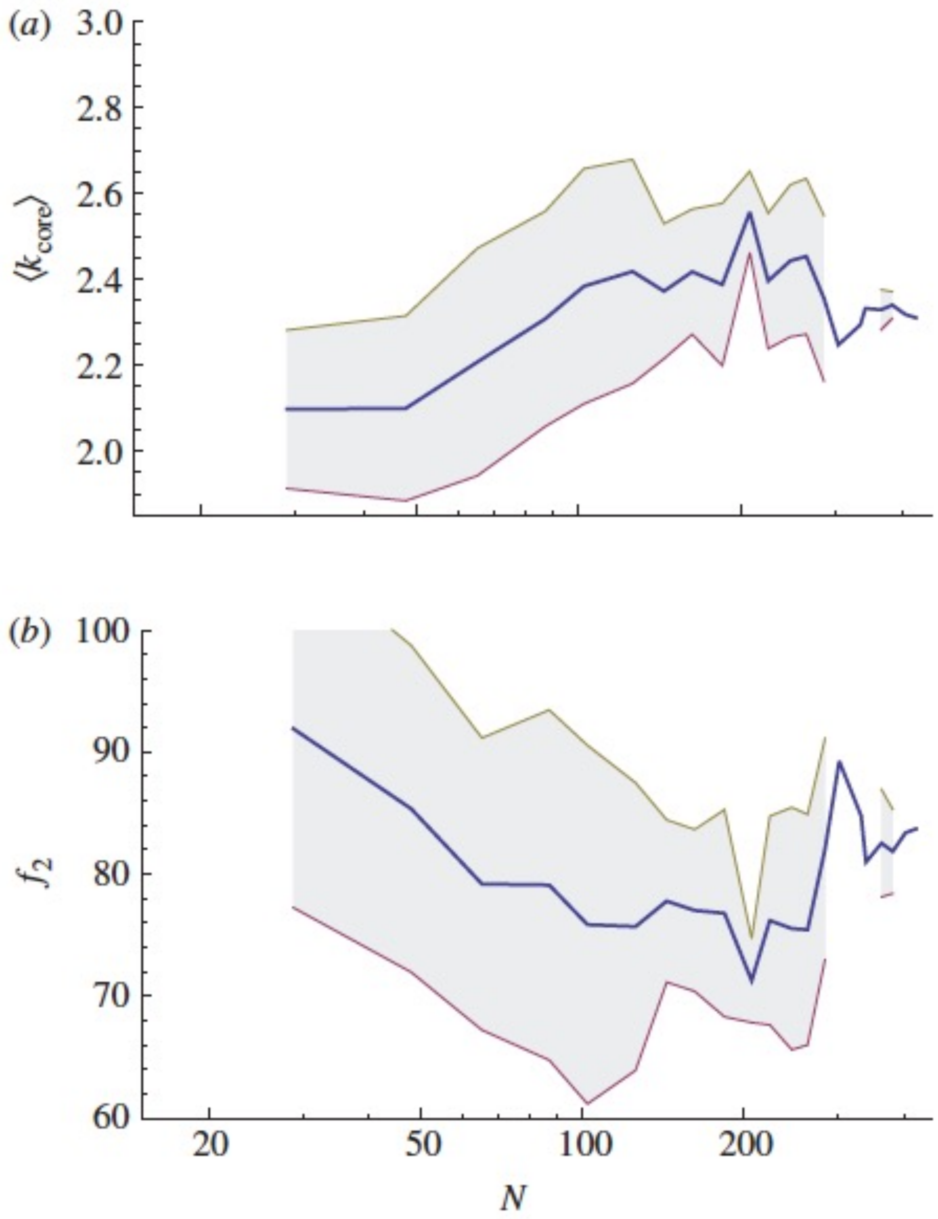


Figure 8. (a) Average degree of the core $\langle k_{core} \rangle$ (equation (3.3)) and its dispersion versus number of stations (averaged over 20 bins). (b) Evolution of the percentage f_2 of $k = 2$ core nodes (averaged over 20 bins). (Online version in colour.)

Spatial extension of branches

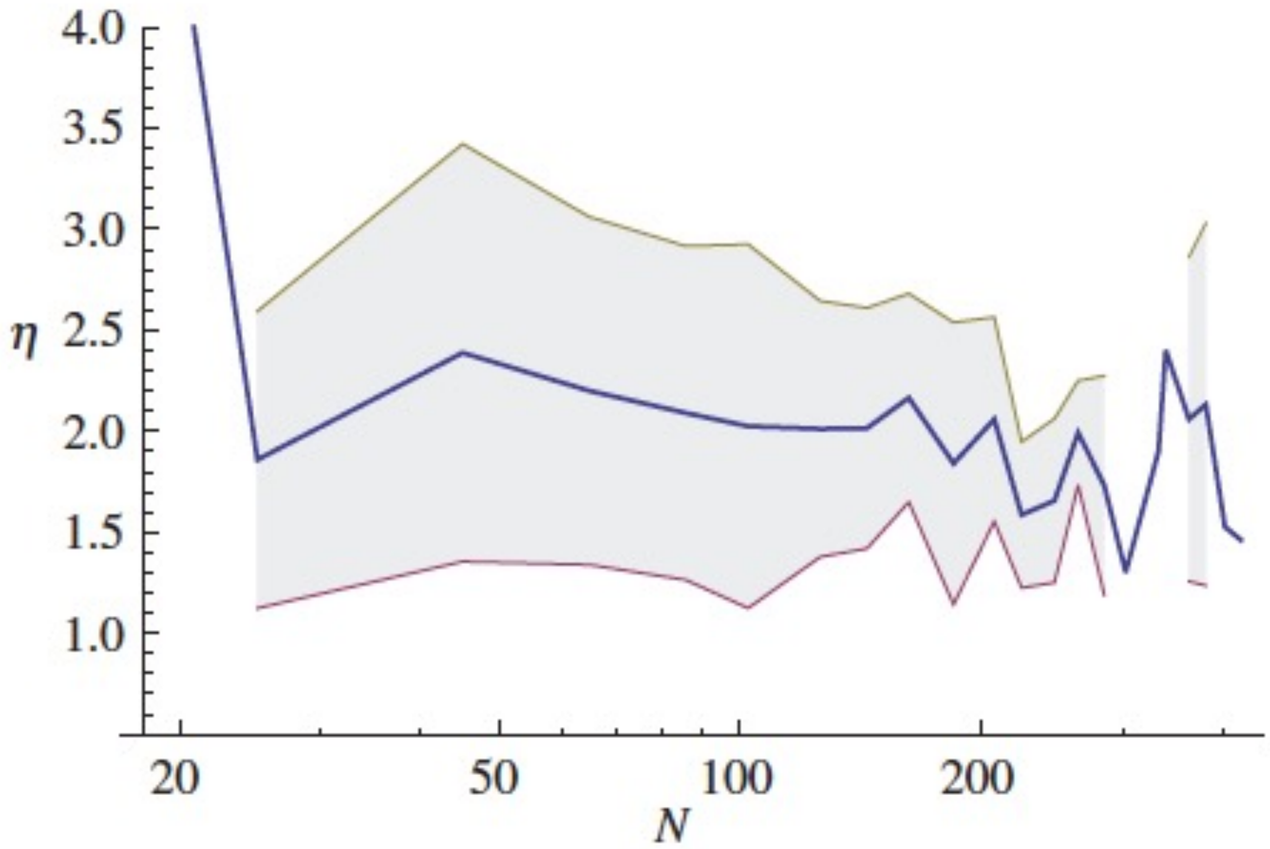


Figure 7. Evolution of the ratio η, which characterizes the spatial extension of branches relative to the core. (Online version in colour.)

Measures on this universal structure

- Characterizing the core

$$\langle k_{core} \rangle = \frac{2E_C(t)}{N_C(t)}$$

$$\alpha(t) = \frac{E_C(t) - N_C(t) + 1}{2N_C(t) - 5}$$

N_C: number of nodes in the core
E_C: number of links in the core

Measures on this universal structure

- Characterizing the branches

$$\beta(t) = \frac{N_B(t)}{N_B(t) + N_C(t)}$$

N_B: number of stations in branches
N_C: number of stations in the core

$$\eta(t) = \frac{\overline{D}_B(t)}{\overline{D}_C(t)}$$

D_B: average distance from barycenter to branches stations
D_C: average distance from barycenter to core stations

Number of branches

- If the spacing b between two branches is constant:

$$\mathcal{N}_B \sim \text{ring perimeter}/b$$

- For a lattice of size N

$$\mathcal{N}_B \sim \sqrt{N}$$

Number of branches

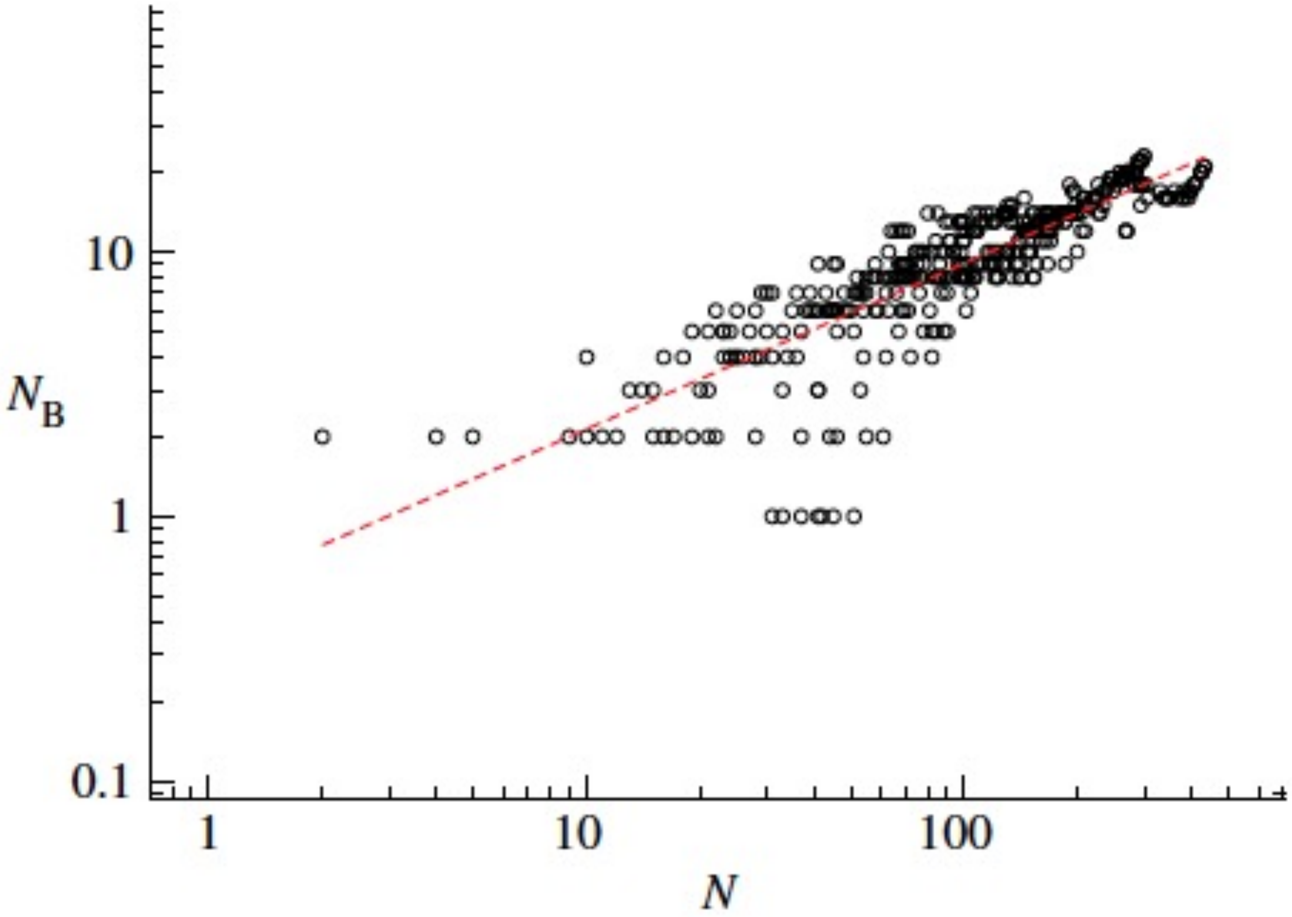
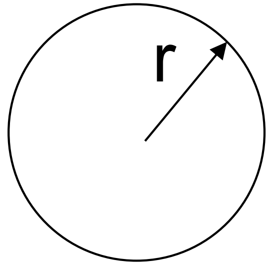


Figure 10. The log–log plot of the number of different branches versus the number of stations for the different subway networks considered here. The dashed line is a power law fit with exponent ≈0.6. (Online version in colour.)

Spatial organization of the core and branches

- Old result for Paris (Benguigui, Daoud 1991)

$$\text{Short scale: } N(r) \sim r^2$$
$$\text{Long scale: } N(r) \sim r^{0.5}$$



N(r): number of stations at distance less than r from barycenter

First regime: homogeneous distribution with d_f=2
Second regime ?

Spatial organization of the core and branches

- Natural explanation with the universal template

$$N(r) = \begin{cases} \rho_C \pi r^2 & \text{for } \; r \ll r_C \\ \rho_C \pi r_C^2 + N_b \int_{r_C}^{r} \frac{dr}{\Delta(r)} & \text{for } r_C \ll r < r_{max} \end{cases}$$

First regime: homogeneous distribution with d_f=2: core
Second regime: branches with increasing inter-stop distance

ρ_C : core density
N_b : number of branches
$\Delta(r)$: Interstation spacing at distance r

Spatial organization of the core and branches

- Interstation spacing at distance r

$$\Delta(r) \sim r^{\tau} \Rightarrow N(r \gg r_C) \sim r^{1-\tau}$$

$$\tau \simeq 0.05 \text{ (Moscow)}$$
$$\tau \simeq 0.5 \text{ (Paris)}$$

- Natural explanation of the Benguigui-Daoud result

Note - Optimal network ?

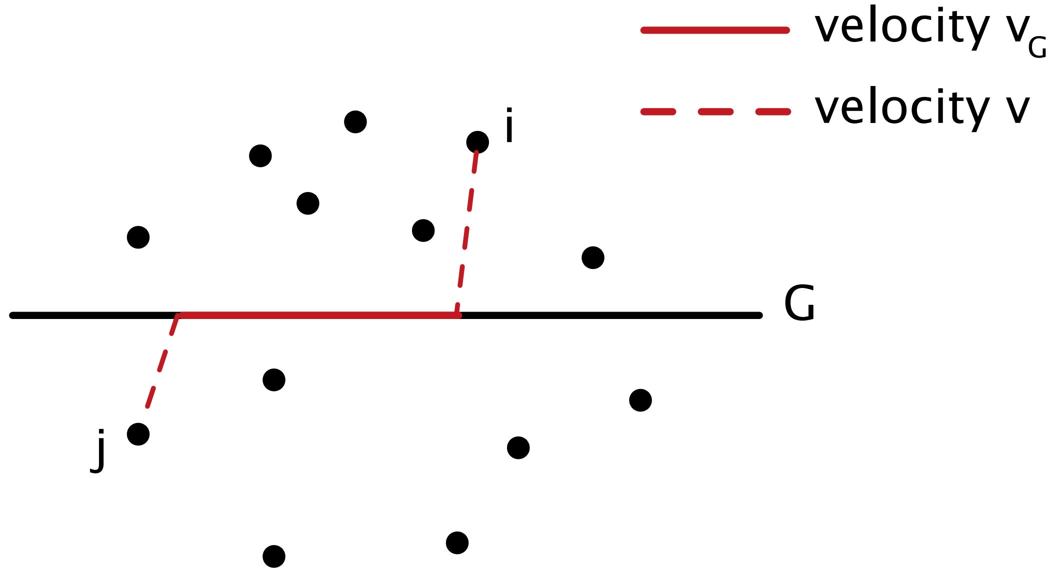
- Optimal graphs on a set of points: MST, etc.

- Here: a different problem ("locational optimization", cf. Okabe). Given a set of points in the plane, find the graph G of length L, such that the average shortest path time

$$\langle \tau \rangle = \frac{1}{\mathcal{N}} \sum_{i,j} \tau(i,j)$$

is minimum. The sum is over all points and the velocity ratio $v_G/v>1$

  - Partial results on simplified variants (with D. Aldous)
  - Optimal graph vs. L: open problem Existence of transitions…



Aldous & MB (2019)

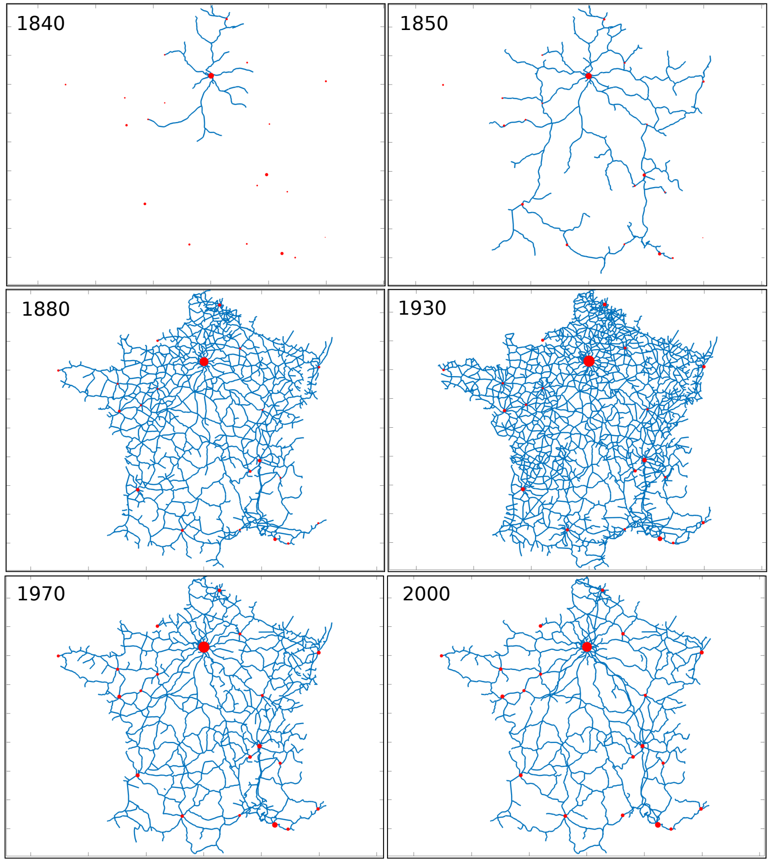
Subways: recap and perspectives

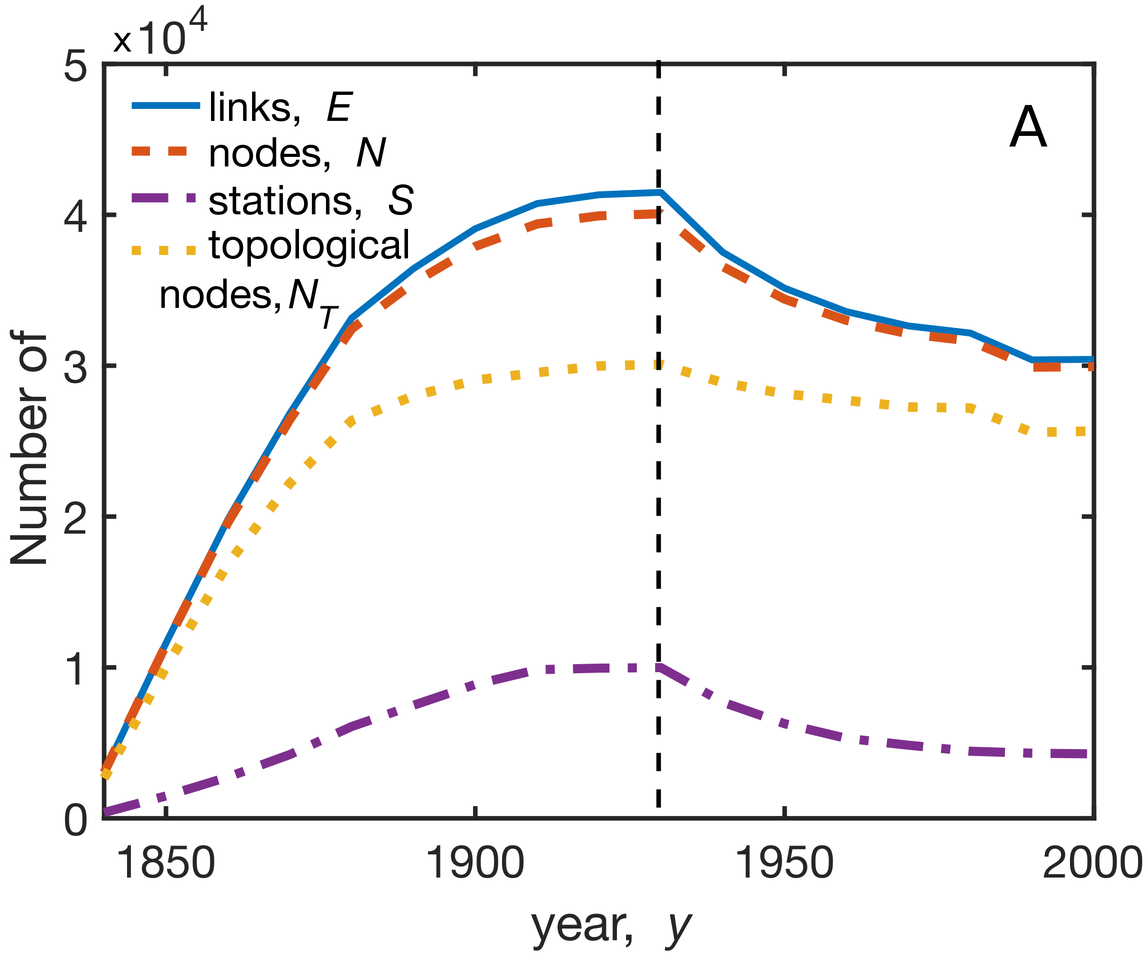
- Convergence to a similar structure made of core and branches
  - Fraction of branches of order 50%
  - Extension of branches/core extension of order 2
  - Average degree of core of order 2.5 and f_2>60%
  - Number of branches scales as $N^{1/2}$
  - Fractal behavior understood within this picture

=> simple model ?

Efficiency and evolution of railway networks

French railway evolution 1840-2000





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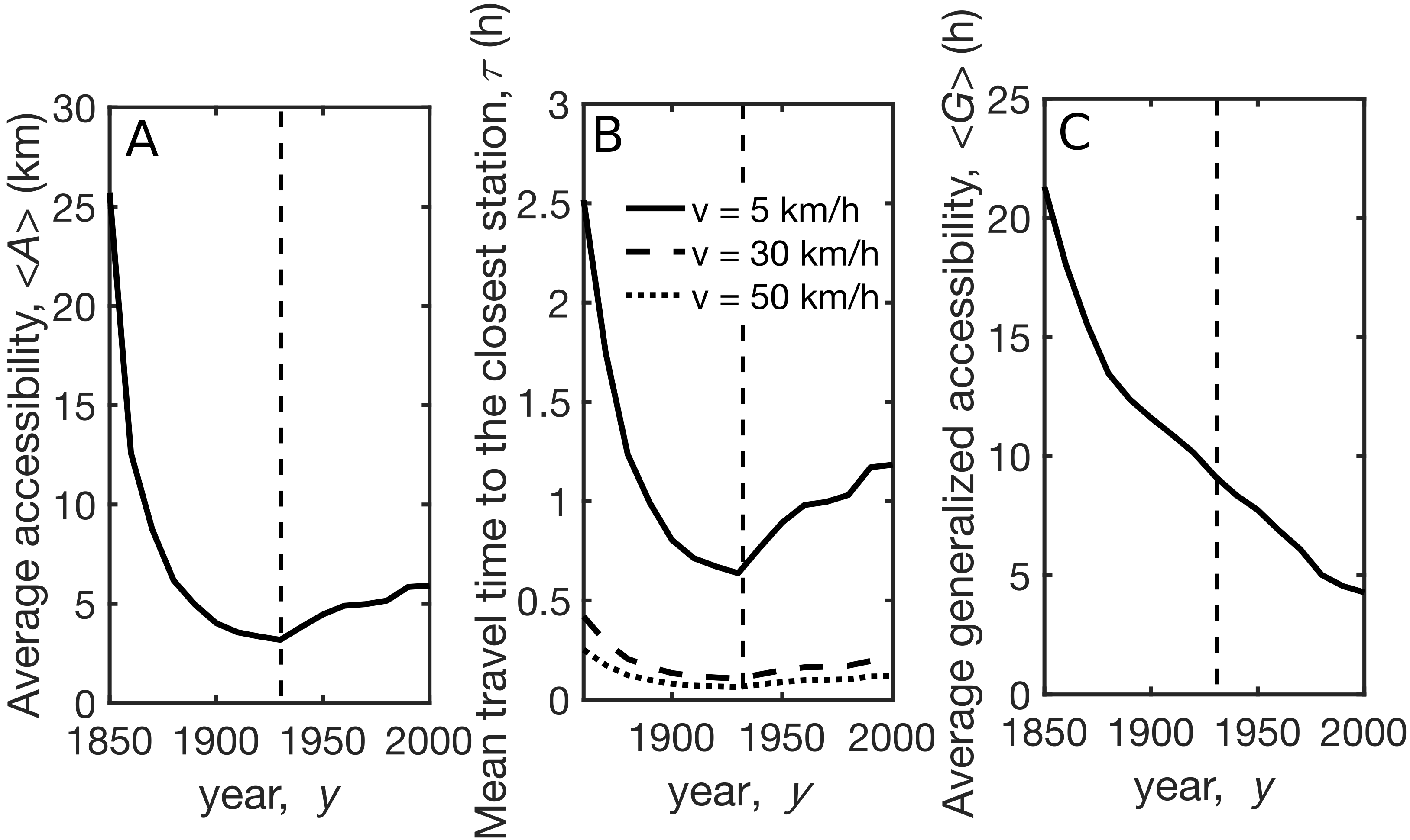
French railway evolution 1840-2000
Coupling with the population evolution



Average distance (time) to nearest station

Average time from any commune to any other: decreasing !

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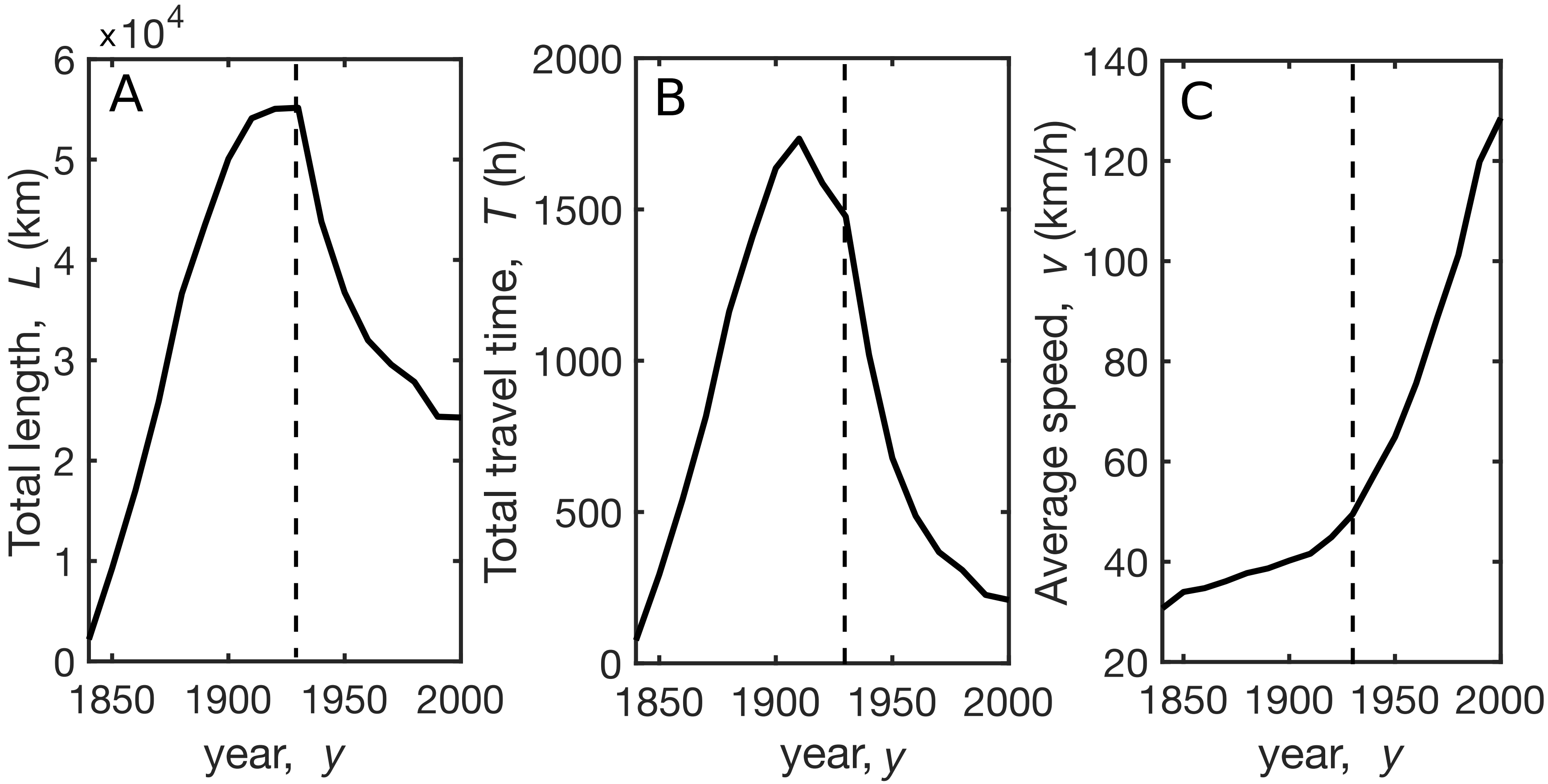
French railway evolution 1840-2000



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Method and goal

- New results on new datasets usually imply to have ..new tools !

- Stylized facts
  - Simplified empirical finding

- Minimal models
  - Explain the largest number of stylized facts with the minimum number of parameters

- Interesting direction
  - Coupling with socio-economical indicators (population, GDP, etc.)

Introduction

- Increasing availability of historical urban data

- From historical maps: evolution of the road structure
  - At the country level (Cassini)
  - Regional level (Groane)
  - City level (Paris)

- Now: from directories => evolution of economic activity (Julie Gravier)

Evolution of transportation networks

- Kansky (1963-69)

Evolution of the Sicilian Railroad network

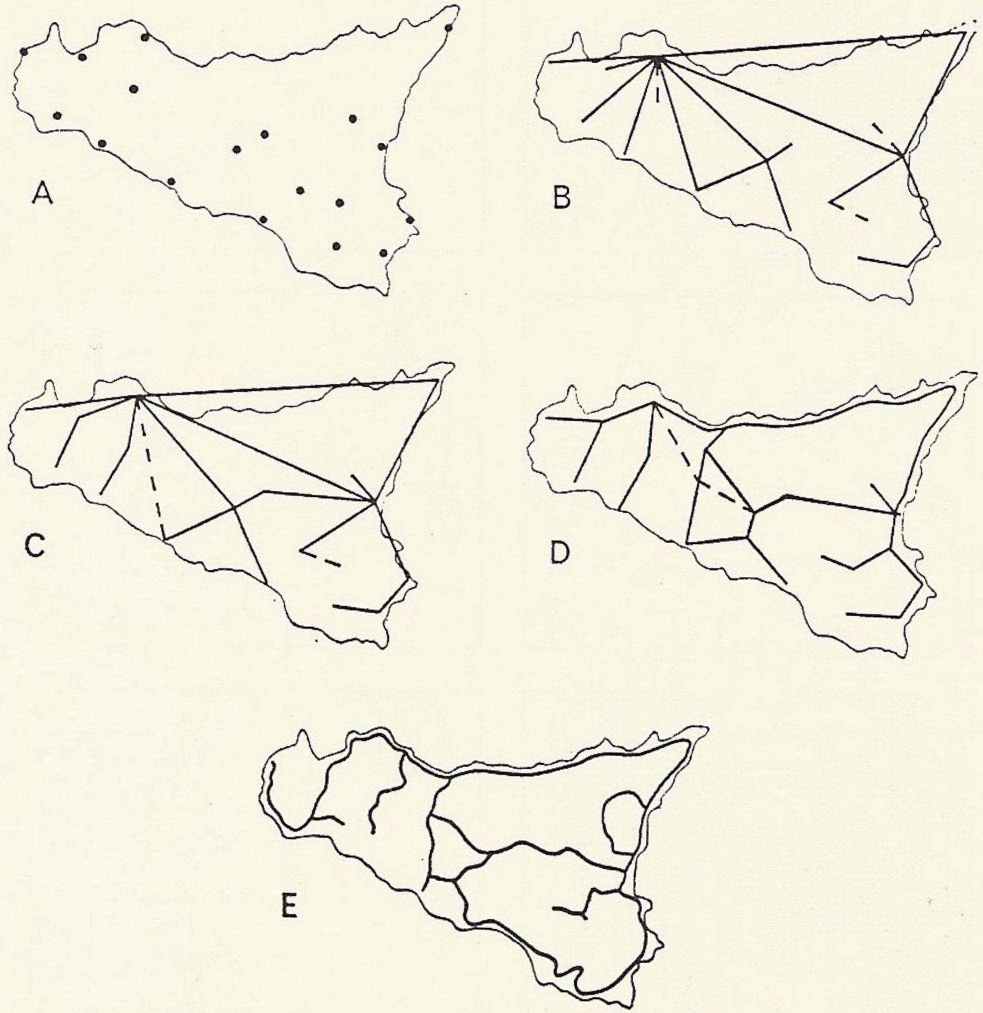


Fig. 5.29. Stages in link allocation for simulating the Sicilian 1908 railroad network. (*A*) Selected vertices. (*B–C*) Stages in link allocation. (*D*) Postdicted 1908 network. (*E*) Actual 1908 network. Source: Kansky, 1963, pp. 139–46.

Evolution of transportation networks

- Morrill (1965)

Railway network growth
(Black, 69)

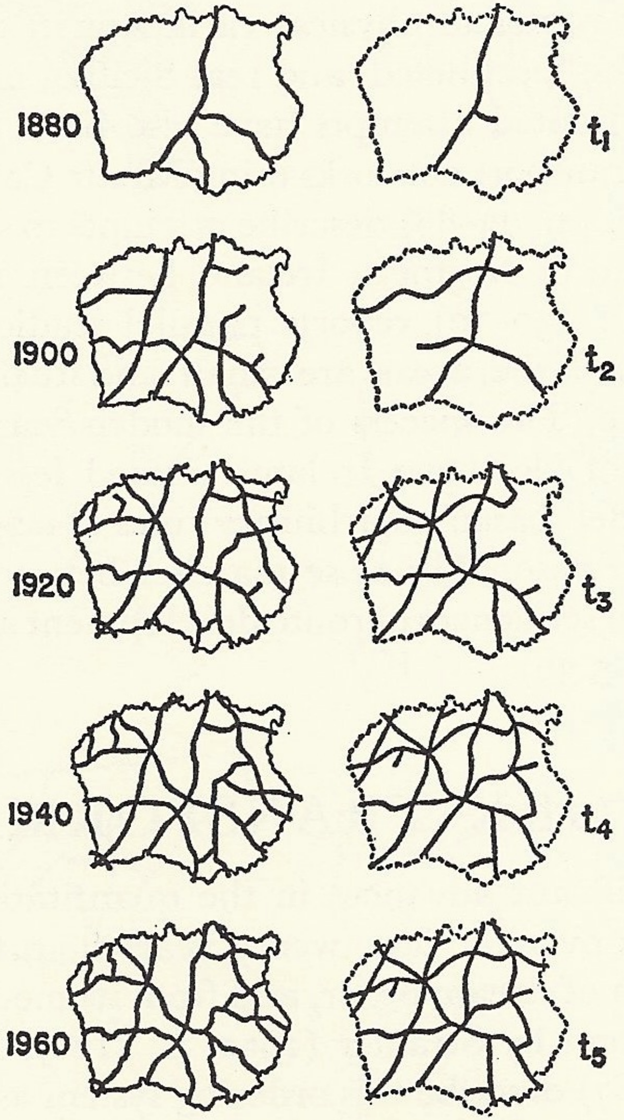


Fig. 5.30. Comparison of the (*A*) actual growth of railway network in central Sweden with (*B*) the simulated growth. Source: Morrill, 1965, pp. 130–70.

Evolution of transportation networks

- Morril (1965)

Road network
stochastic evolution

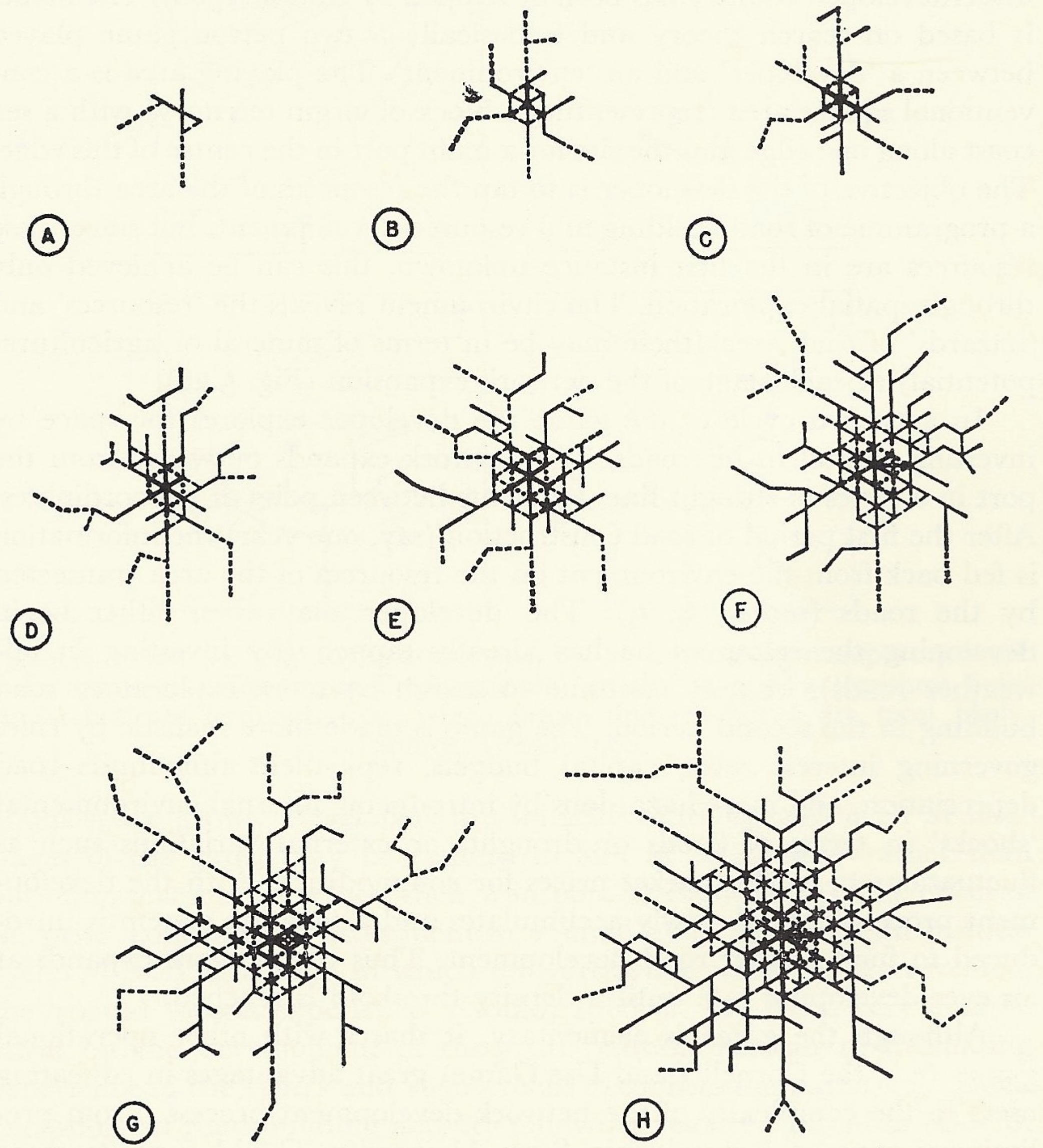


Fig. 5.25. Simulation of road network around a central node by Monte Carlo procedures. Source: Morrill, 1965.

Simple measures

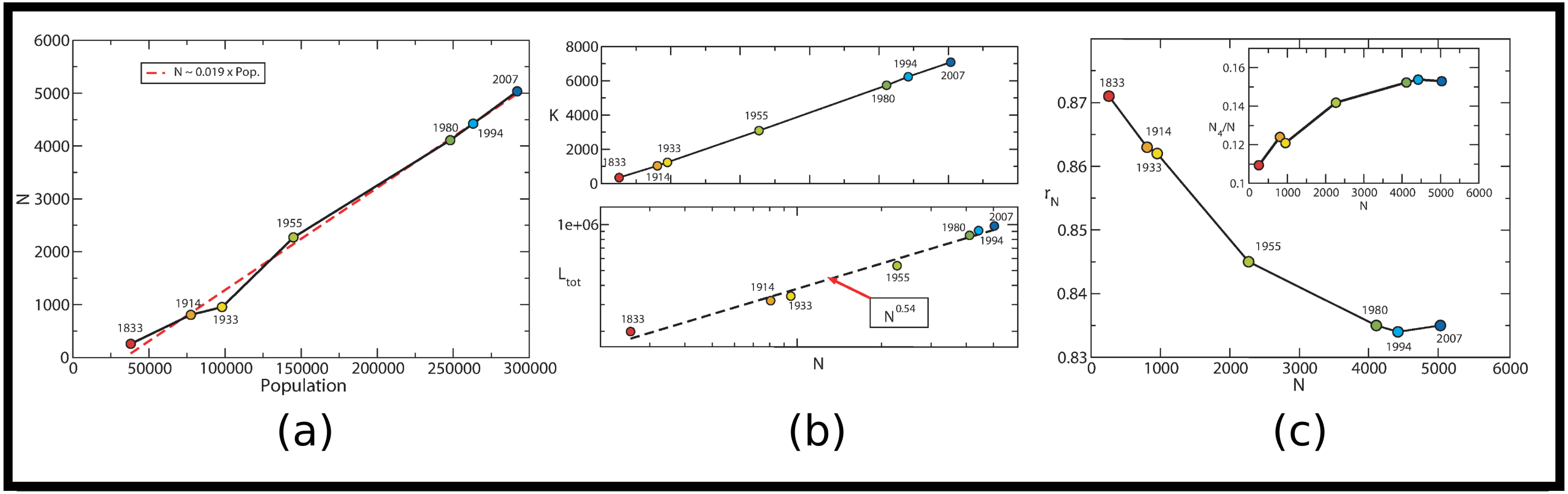
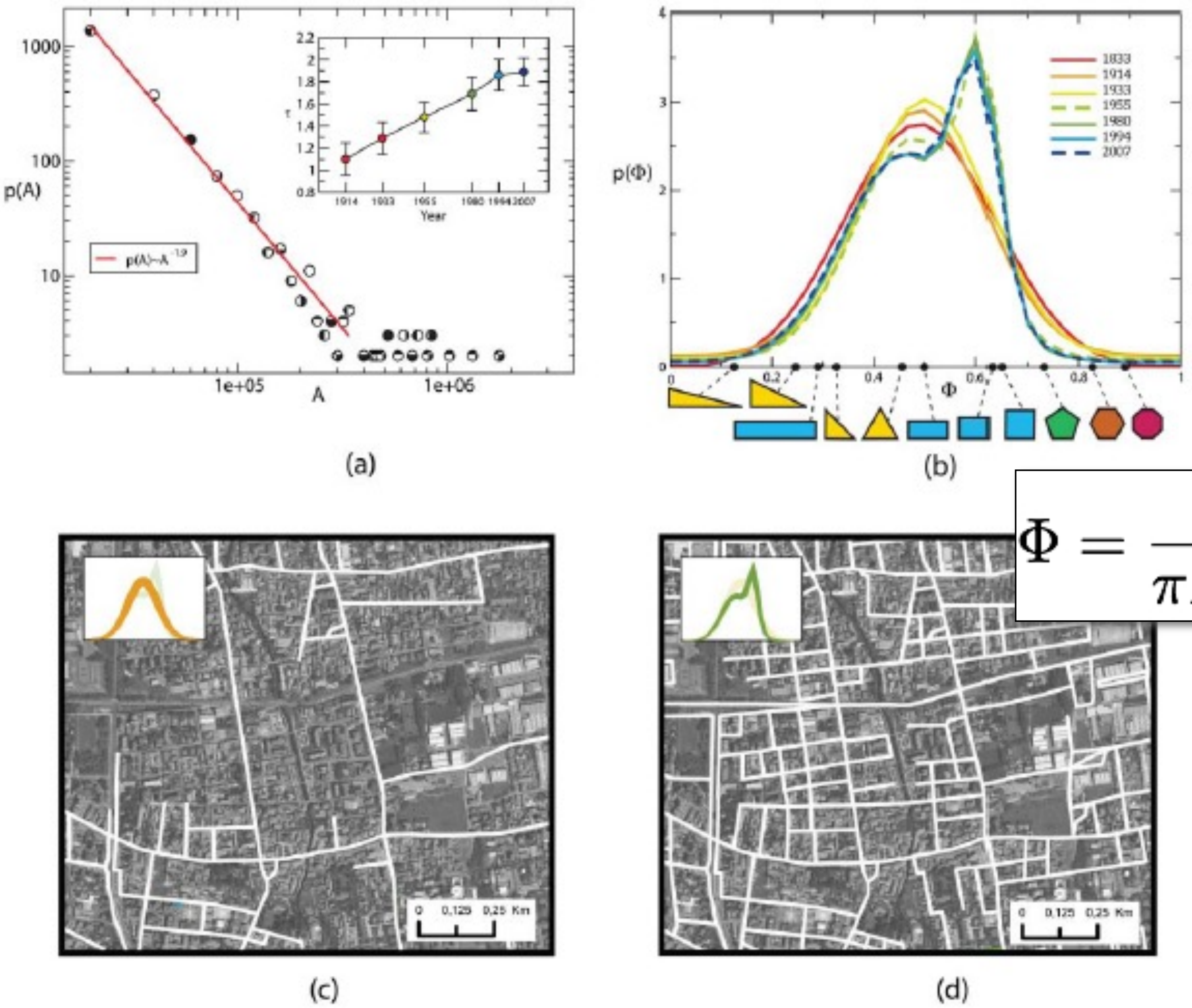


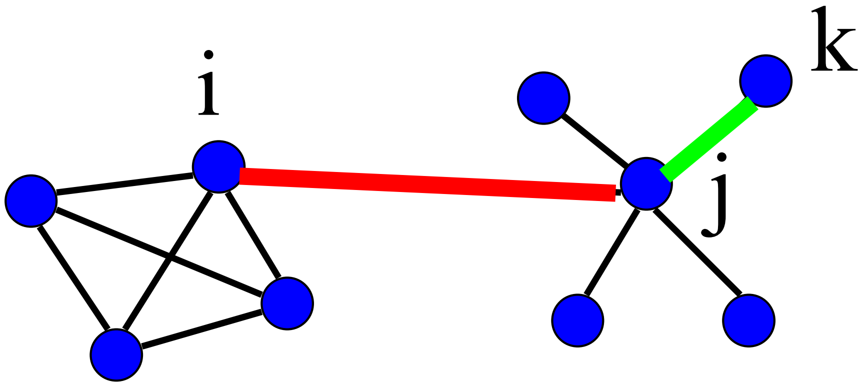
Figure 2 | (a) Number of nodes N versus total population (continuous line with circles) and its linear fit (red dashed line). (b) Total number of links E and total network length L_{tot}, as a function of the number of nodes N. The total network length increases as $N^{0.54}$. (c) Value of the ratio r_N between the number of nodes with degree $k = 1$ and $k = 3$, and the total number of nodes. In the inset we report the percentage of nodes having degree $k = 4$ as a function of N. Notice that the relative abundance of four-ways crossings increases by 5% in two centuries.

$$\frac{dN}{dP} \simeq 0.019 \qquad \ell_1 \sim 1/\sqrt{\rho} \Rightarrow L_{tot} \sim \sqrt{N} \qquad r_N = \frac{N(1) + N(3)}{\sum_{k \neq 2} N(k)}$$



$$\Phi = \frac{A}{\pi D^2/4}$$

Betweenness Centrality (Freeman '77)



ij: large centrality

jk: small centrality

$$b(ij) = \sum_{s,t} \frac{\sigma_{st}(ij)}{\sigma_{st}}$$

σ_{st} = # of shortest paths from s to t

$\sigma_{st}(ij)$= # of shortest paths from s to t **via** (ij)

New links: characterization

- Characterization of new links: BC impact

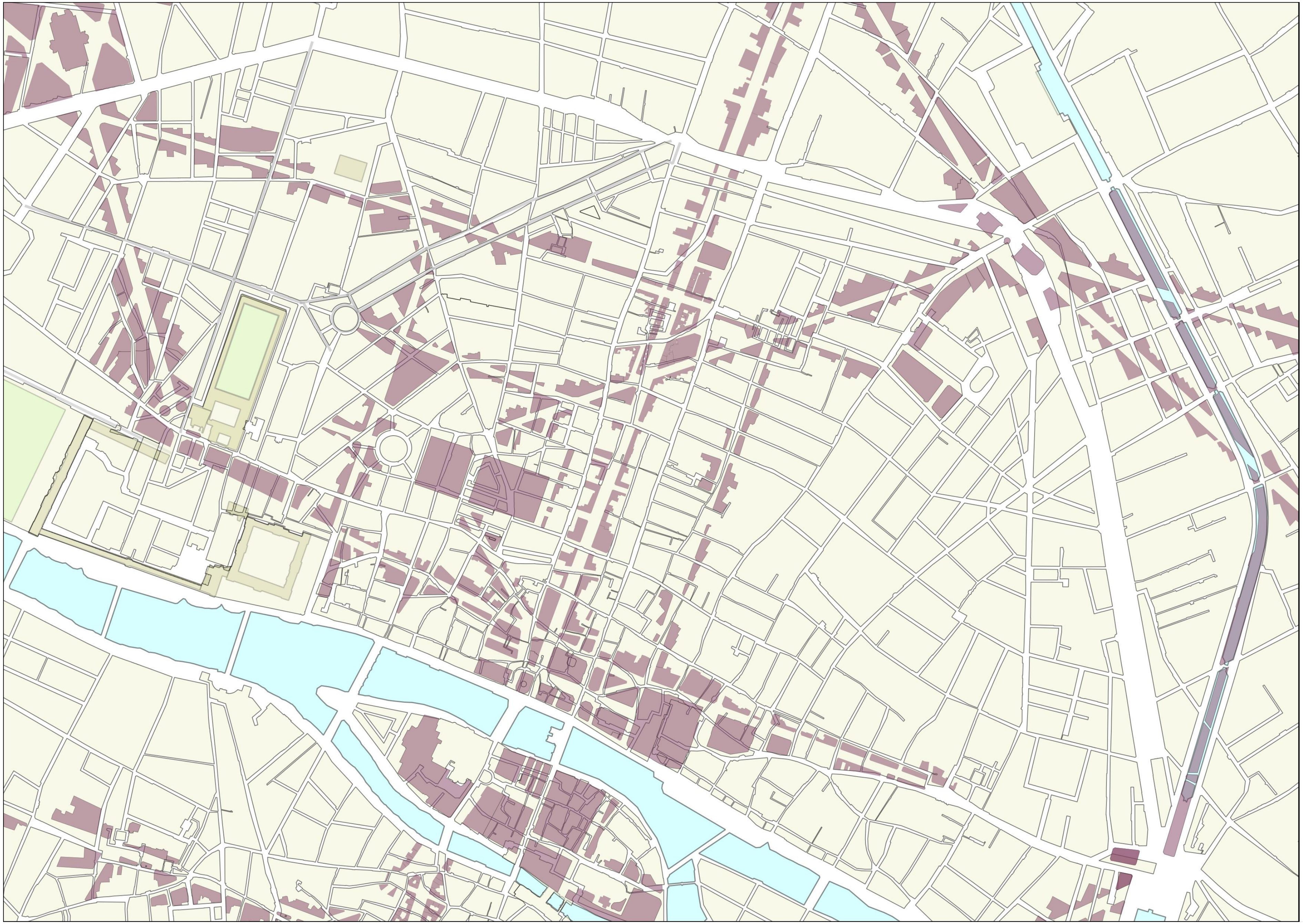
$$\bar{b}(G_t) = \frac{1}{(N(t)-1)(N(t)-2)} \sum_{e \in E_t} b(e)$$

$$\delta b(e*) = \frac{\bar{b}(G_t) - \bar{b}(G_t \setminus \{e*\})}{\bar{b}(G_t)}$$

Summary

- Homogeneization of cell shapes
- Existence of a stable backbone of central roads
- 2 main elementary processes: densification and exploration
- Modelling: fragmentation models ?

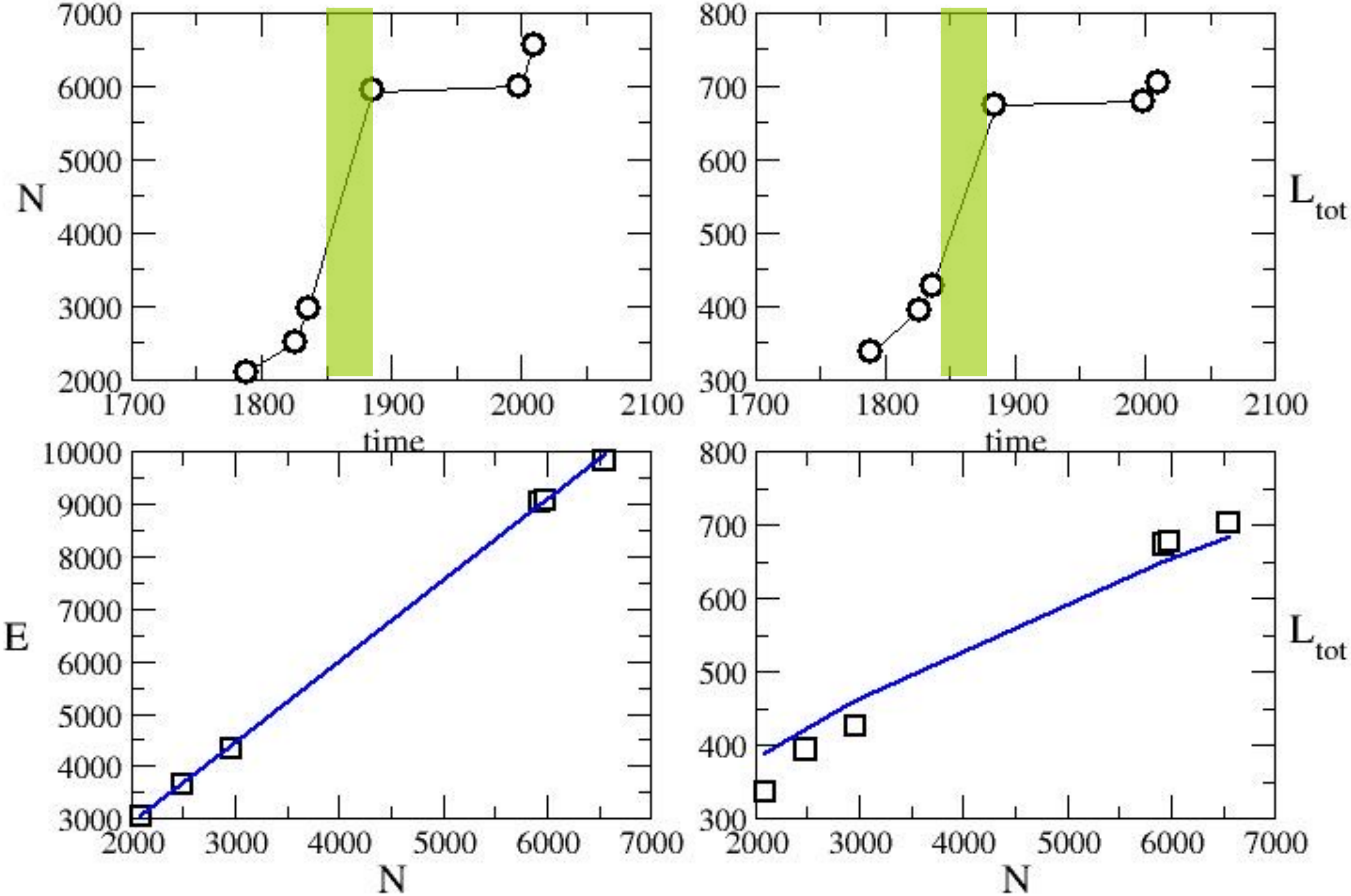
Importance of central planning



Importance of central planning

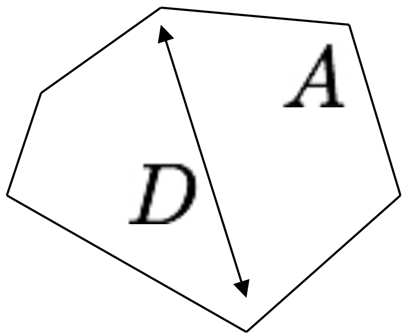
- Standard indicators versus time or N

$$\frac{\Delta L_{tot}}{\Delta t} \approx 1.6 \;\; \text{km/year}$$

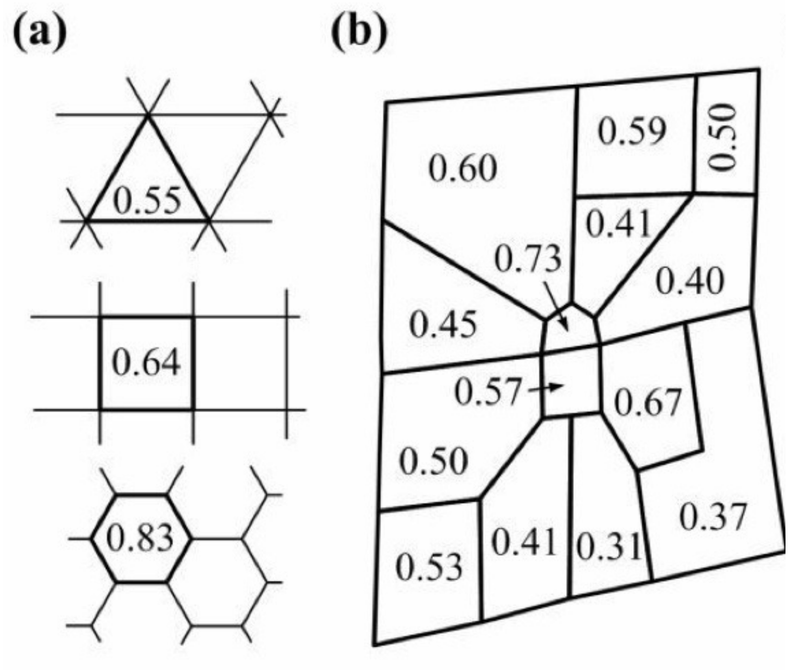


$$L_{tot} \sim \frac{\langle k \rangle}{2} \sqrt{AN}$$

Cell (block, face) shape



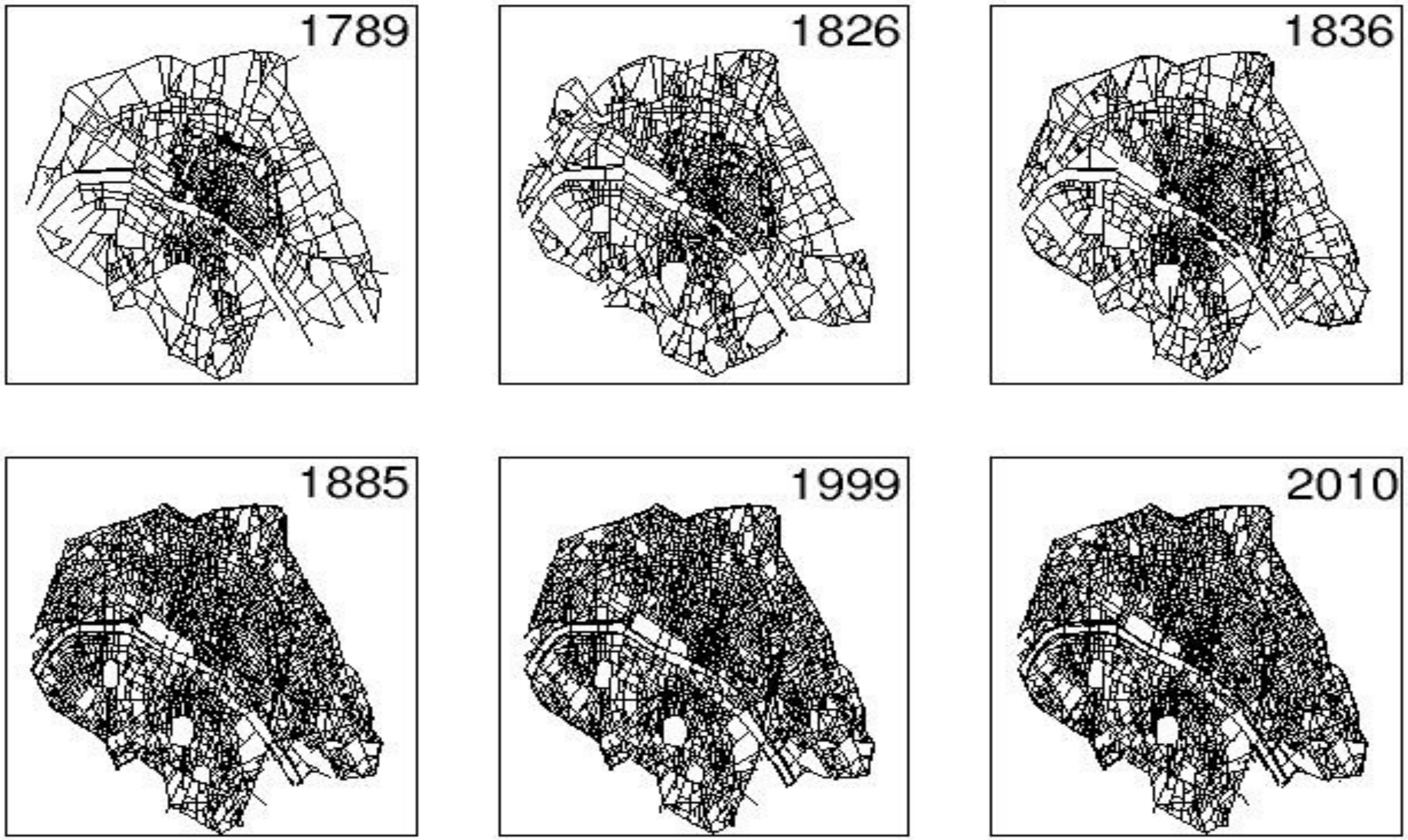
$$\phi = \frac{A}{(\pi D^2/4)}$$



Importance of planning

- Portion defined by Paris in 1789

before Haussmann



after Haussmann

World subway networks

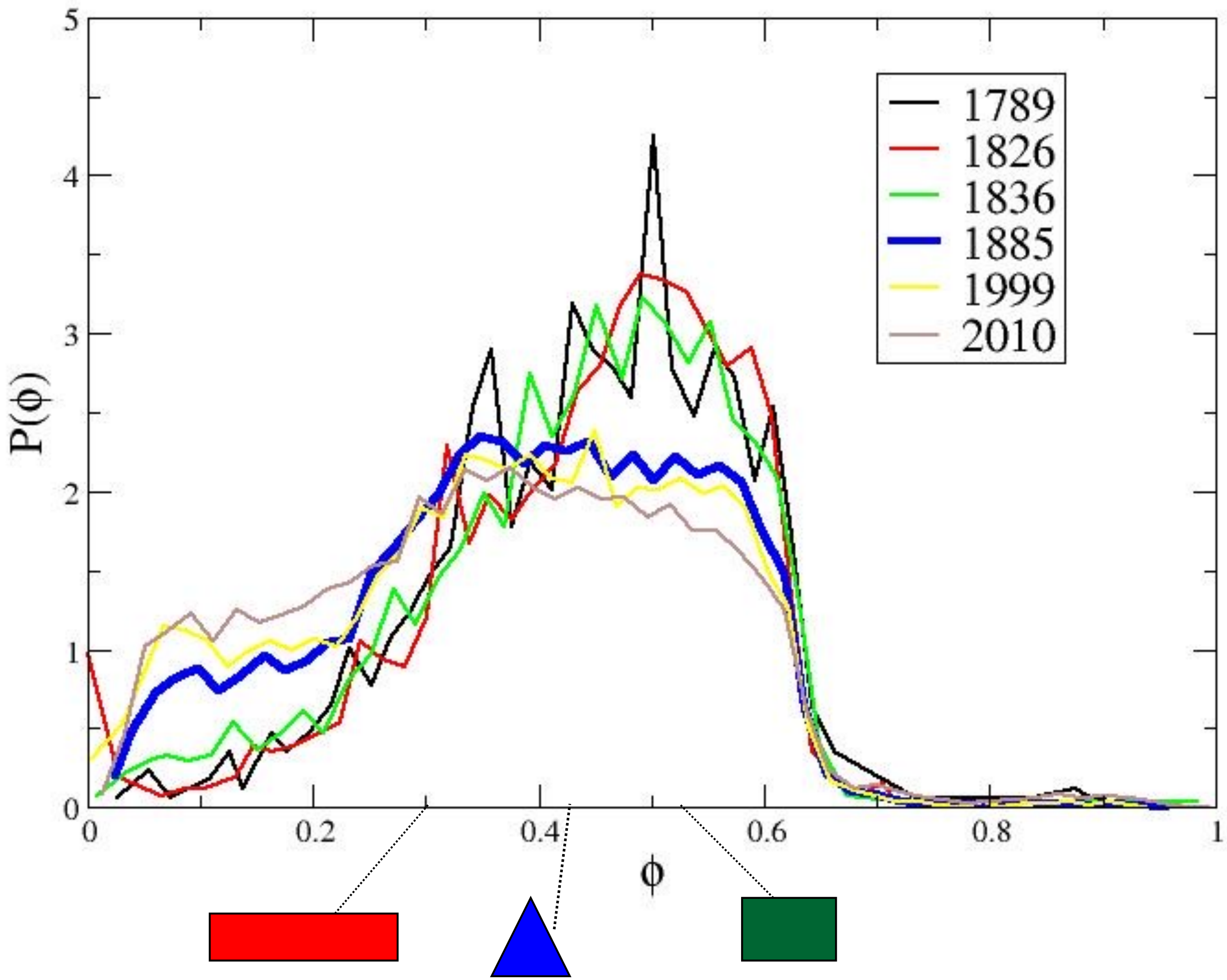
- Some simple properties

city	t_0	$\bar{v}$	σ_v	f (%)
Beijing	1971	3.3	7.74	79
Tokyo	1927	2.8	5.47	51
Seoul	1974	11.2	14.9	20
Paris	1900	2.6	5.1	60
Mexico City	1969	3.7	5.9	55
NYC	1878	3.3	8.3	68
Chicago	1901	1.9	6.24	71
London	1863	2.3	3.8	48
Shanghai	1995	14.9	20.2	31
Moscow	1936	1.7	1.9	43
Berlin	1901	1.6	3.3	65
Madrid	1919	2.3	4.6	59
Osaka	1934	1.4	4.1	79
Barcelona	1914	1.4	4.8	78

- (i) large velocities at small times; (ii) large fluctuations from year to year; (iii) inactivity f on average 60%

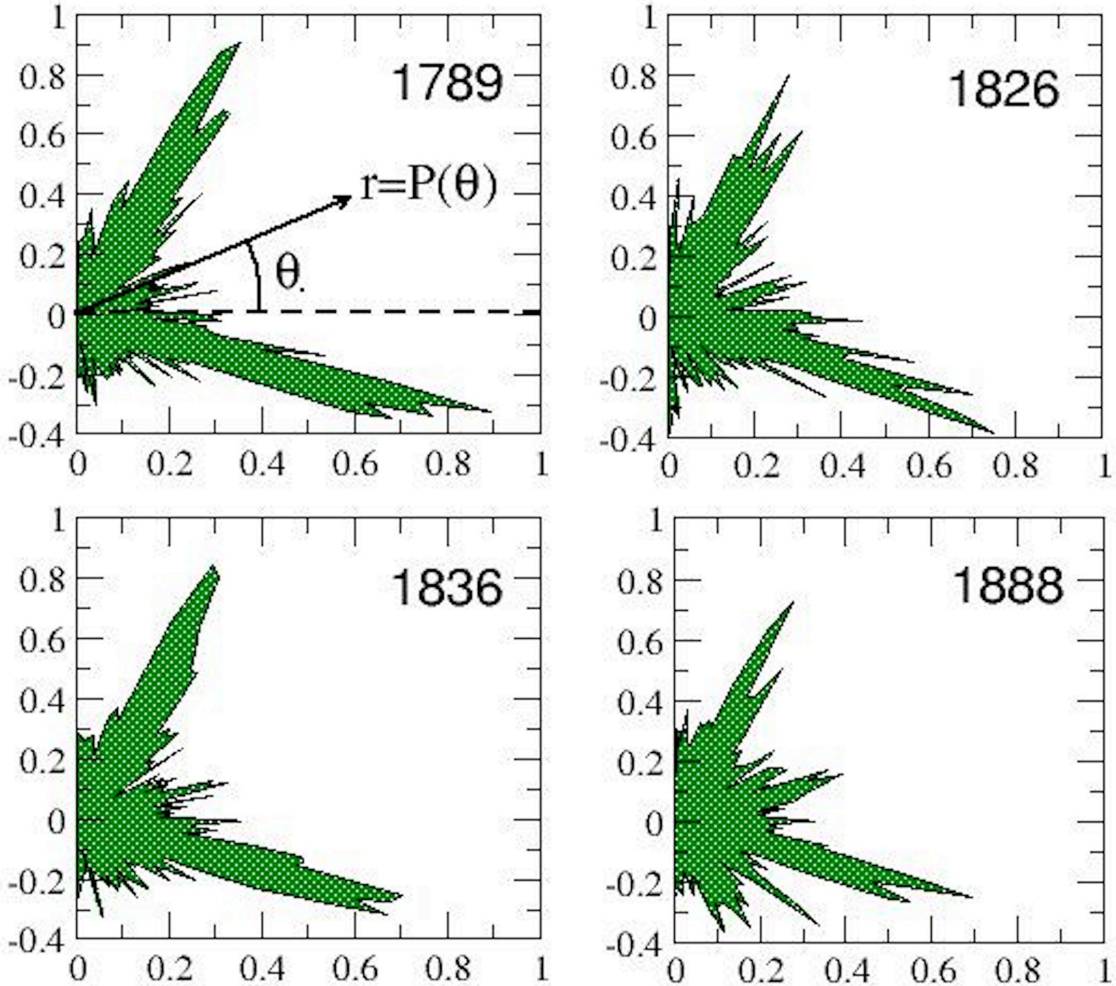
Haussmann effect

- Shape factor



Importance of central planning

- Angle distribution



Incidentally: angle distribution

Figure 3. Street networks and corresponding polar histograms for Manhattan and Boston.

G. Boeing, arXiv:1808.00600 (2018): OSM and OSMnx

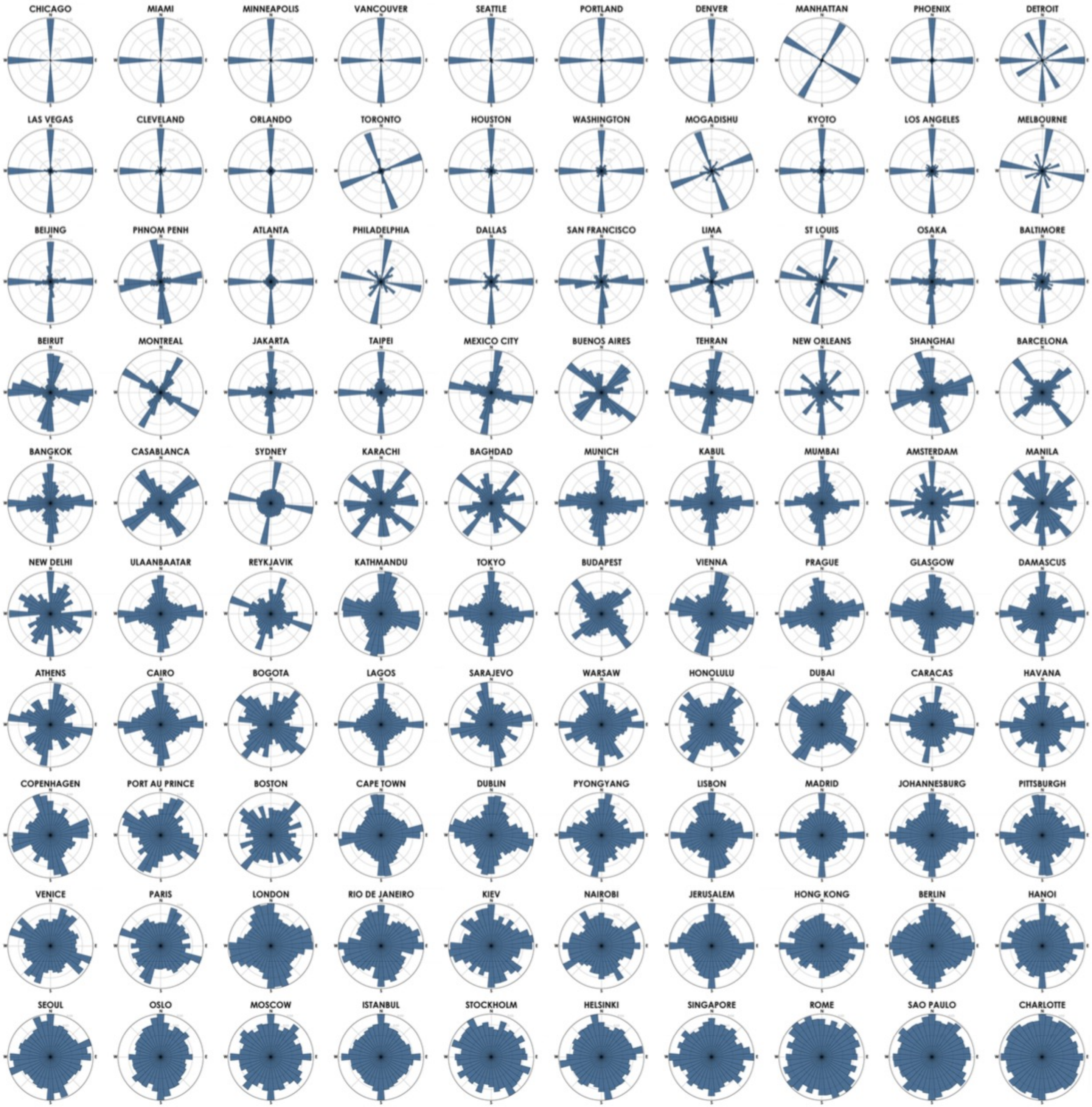
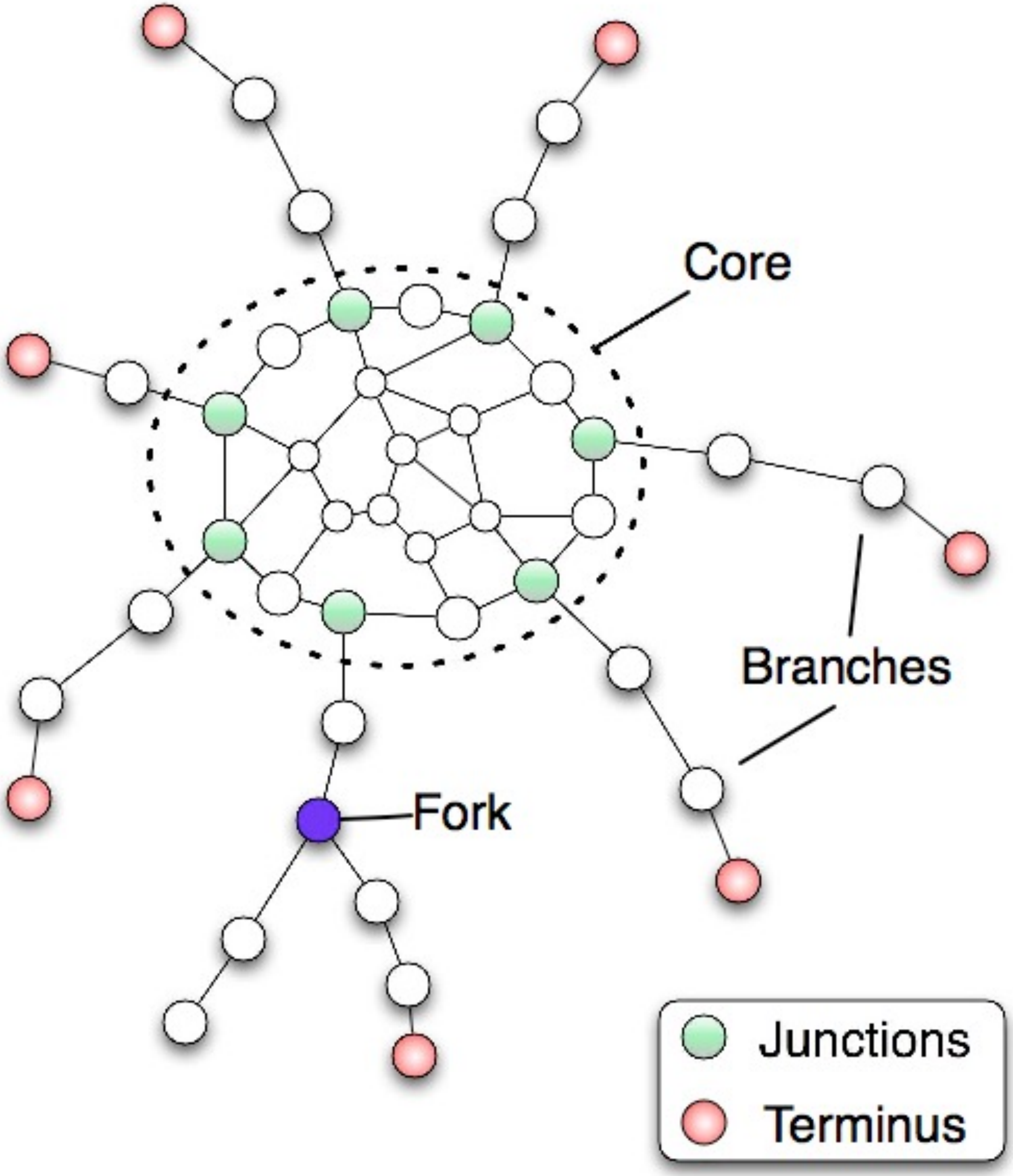


Figure 5. Polar histograms of 100 world cities' street orientations, sorted by descending ρ from most to least grid-like

Universal template

- 3 elements
  - Core
  - Ring
  - Branches



# Road network China
1600(BC)- 1900 (AC)

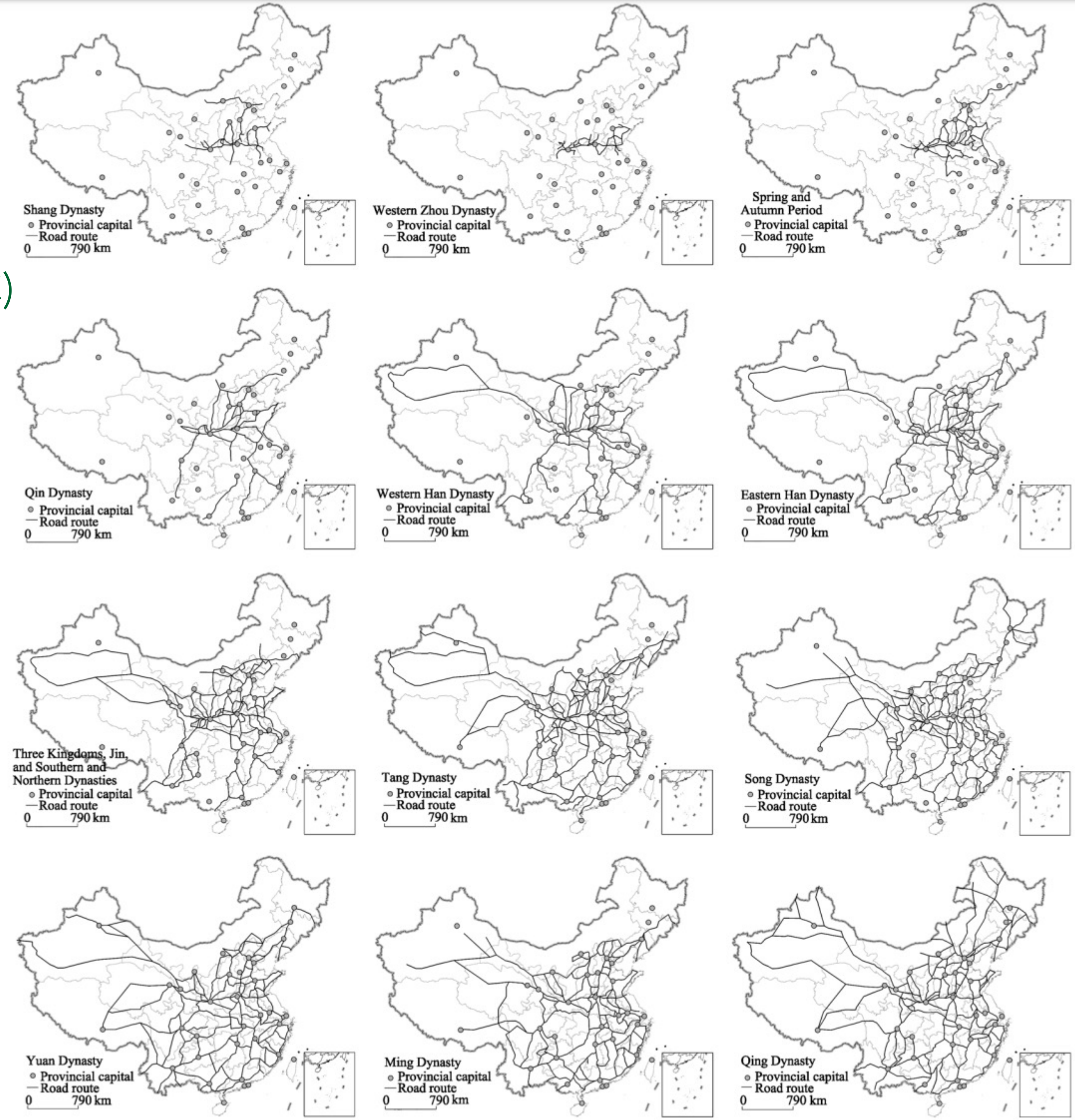


Figure 1 Spatial evolution of the road network in China since the Shang Dynasty

Wang, Ducruet, Wang (2015)